Problem 2.

1. Assume a, b, and c are odd integers
2. $a = 2i + 1$ for some integer i
3. $b = 2j + 1$ for some integer j
4. $c = 2k + 1$ for some integer k
5. $a + b + c = 2i + 1 + 2j + 1 + 2k + 1$
6. $a + b + c = 2i + 2j + 2k + 2 + 1$
7. $a + b + c = 2(i + j + k + 1) + 1$ where $i + j + k + 1$ is an integer
8. $a + b + c$ is an odd integer
9. if a, b, and c are odd integers, then $a + b + c$ is an odd integer

Problem 3.

a.

1. Assume x is rational
2. $x = \frac{p}{q}$ where p and q are integers and $q \neq 0$
3. $x - 5 = \frac{p}{q} - 5$
4. $x - 5 = \frac{p}{q} - \frac{5q}{q}$
5. $x - 5 = \frac{p-5q}{q}$ where $p - 5q$ is an integer
6. $x - 5$ is rational
7. If x is rational then $x - 5$ is rational

b.

1. Assume $x - 5$ is rational
2. $x - 5 = \frac{p}{q}$ where p and q are integers and $q \neq 0$
3. $x = \frac{p}{q} + 5$
4. $x = \frac{p}{q} + \frac{5q}{q}$
5. $x = \frac{p + 5q}{q}$
6. $x/3 = \frac{p+5q}{3q}$ where $p + 5q$ and $3q$ are integers and $3q \neq 0$
7. $x/3$ is rational
8. If $x - 5$ is rational then $x/3$ is rational

c.

1. Assume $x/3$ is rational
2. $x/3 = \frac{p}{q}$ where p and q are integers and $q \neq 0$
3. $x = \frac{3p}{q}$ where $3p$ and q are integers
4. x is rational
5. If $x/3$ is rational then x is rational

Problem 4.

a. Proof by contrapositive

1. Assume that it is not the case that m is odd or n is odd for a proof by contraposition
2. m is even and n is even (by De Morgan)
3. $m = 2i$ for some integer i and $n = 2j$ for some integer j
4. $m - n = 2i - 2j$
5. $m - n = 2(i - j)$ where $i - j$ is an integer
6. $m - n$ is even
7. $m - n$ is not odd

8. If it is not the case that m is odd or n is odd then $m - n$ is not odd
9. If $m - n$ is odd, then m is odd or n is odd

b. Proof by contradiction

1. Assume that it is not the case that if $m - n$ is odd, then m is odd or n is odd
2. $m - n$ is odd and it is not the case that m is odd or n is odd
3. $m - n$ is odd, m is not odd and n is not odd (by De Morgan)
4. $m - n$ is odd, m is even and n is even
5. $m - n$ is odd, $m = 2i$ and $n = 2j$ for integers i and j
6. $m - n$ is odd and $m - n = 2i - 2j$
7. $m - n$ is odd and $m - n = 2(i - j)$ where $i - j$ is an integer
8. $m - n$ is odd and $m - n$ is even
9. This is a contradiction because a number cannot be both odd and even
10. Therefore, if $m - n$ is odd, then m is odd or n is odd