Homework Assignment 9
CS 2233
Section 001 and Section 002
Due: Friday, April 26

Problem 1. [10 points]
Complete all participation activities in zyBook sections 6.1-6.3

Problem 2. [25 points]
a. [5 points] List all of the ordered pairs in the relation $R = \{(a, b) |\ a + b \text{ is even}\}$ on the set $\{1, 2, 3, 4, 5\}$

$R = \{(1,1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (5, 5)\}$

b. [10 points] For each of the following relations on the set of all people, determine if the relation is necessarily reflexive, symmetric, antisymmetric, and/or transitive

1) [5 points] $R_1 = \{(a, b) | \text{There is a book that both } a \text{ and } b \text{ have read}\}$
Is not reflexive, is symmetric, is not antisymmetric, is not transitive

2) [5 points] $R_2 = \{(a, b) | a \text{ has a higher gpa than } b\}$
Is not reflexive, is not symmetric, is antisymmetric, is transitive

c. [10 points] For each of the following relations on the set of real numbers, determine if the relation is reflexive, symmetric, antisymmetric, and/or transitive

1) [5 points] $R_3 = \{(x, y) | x - y = 0\}$
Is reflexive, is symmetric, is antisymmetric, is transitive

2) [5 points] $R_4 = \{(x, y) | x < y \vee y < x\}$
Is not reflexive, is symmetric, is not antisymmetric, is not transitive

Problem 3. [25 points] Let F be the set of all functions from integers to integers. For each of the following relations on F, determine if the relation is reflexive, symmetric, and/or transitive.

a. [5 points] $\{(f, g) | f(1) = g(1)\}$
Is reflexive, is symmetric, is transitive

b. [5 points] $\{(f, g) | f(0) = g(0) \text{ or } f(1) = g(1)\}$
Is reflexive, is symmetric, is not transitive

c. [5 points] $\{(f, g) | f(x) - g(x) = 1 \text{ for all } x \in \boldsymbol{Z}\}$
Is not reflexive, is not symmetric, is not transitive

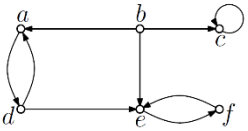
d. [5 points] $\{(f, g) | \text{ for some } C \in Z, \text{for all } x \in Z, f(x) - g(x) = C\}$
Is reflexive, is symmetric, is transitive

e. [5 points] $\{(f, g) | f(0) = g(1) \text{ and } f(1) = g(0)\}$
Is not reflexive, is symmetric, is not transitive

Problem 4. [5 points] List the 4-tuples that are in the relation $\{(a, b, c, d) | a, b, c, d \in \boldsymbol{Z}^+ \text{ and } abcd = 6\}$

$\{(1,1,1,6), (1,1,6,1), (1,6,1,1), (6,1,1,1), (1,1,2,3), (1,1,3,2), (1,2,1,3), (1,2,3,1), (1,3,1,2), (1,3,2,1),$
$(2,1,1,3), (2,1,3,1), (2,3,1,1), (3,1,1,2), (3,1,2,1), (3,2,1,1)\}$

Problem 5. [30 points] For the following directed graph:



a. [5 points] Specify the set of vertices
$V = \{a, b, c, d, e, f\}$

b. [5 points] Specify the set of edges.
$E = \{(a, d), (b, a), (b, c), (b, e), (c, c), (d, a), (d, e), (e, f), (f, e)\}$

c. [5 points] Give the in-degree of each vertex
$\text{in-degree}(a) = 2$
$\text{in-degree}(b) = 0$
$\text{in-degree}(c) = 2$
$\text{in-degree}(d) = 1$
$\text{in-degree}(e) = 3$
$\text{in-degree}(f) = 1$

d. [5 points] Give the out-degree of each vertex
$\text{out-degree}(a) = 1$
$\text{out-degree}(b) = 3$
$\text{out-degree}(c) = 1$
$\text{out-degree}(d) = 2$
$\text{out-degree}(e) = 1$
$\text{out-degree}(f) = 1$

e. [5 points] Is there a path from vertex a to vertex f?
Yes

f. [5 points] Is there a path from vertex f to vertex a?
No