Problem 2. [15 points]
Let the domain of discourse be penguins and the predicate $E(x)$ be true exactly when x eats fish. Express each of the following statements in English

a. [5 points] $\forall x E(x)$
Every penguin eats fish.

b. [5 points] $\neg\exists x E(x)$
There is no penguin that eats fish.

c. [5 points] $\exists x \neg E(x)$
There is a penguin that does not eat fish.

Problem 3. [15 points]
Let the domain of discourse be the set of integers, $\{\cdots-2,-1,0,1,2,\cdots\}$, and let $Q(x)$ be true exactly when $2^x > 3x$. Determine the truth value of each of the following.

a. [5 points] $Q(2)$
False

b. [5 points] $Q(4)$
True

c. [5 points] $\exists x(Q(x) \wedge x < 4)$
True. (When x is 0, $Q(x) \wedge x < 4$ is true)

Problem 4. [10 points]
Let the domain of discourse be all UTSA students, $P(x)$ be true exactly when x understands propositional logic, and $M(x)$ be true exactly when x has taken discrete math. Express the following in English.

a. [5 points] $\forall x(M(x) \rightarrow P(x))$
Every UTSA student who has taken discrete math understands propositional logic

b. [5 points] $\exists x(P(x) \wedge \neg M(x))$
There is a UTSA student who understands propositional logic but has not taken discrete math

Problem 5. [55 points] Let the domain of discourse be college students at a park, let $J(x)$ be true exactly when x is a junior, and let $S(x,y)$ be true exactly when x saw y at the park. Express each of the following in predicate logic.

a. [5 points] x saw someone at the park.
$\exists z\, S(x,z)$

b. [5 points] x was seen at the park.
$\exists z\, S(z,x)$

c. [5 points] x saw all juniors at the park.
$\forall z\, (J(z) \rightarrow S(x,z)$

d. [5 points] x saw only juniors at the park.
$\forall z\, (S(x,z) \rightarrow J(z))$

e. [5 points] All juniors saw x at the park.
$\forall z\, (J(z) \rightarrow S(z,x))$

f. [5 points] Only juniors saw x at the park.
$\forall z\, (S(z,x) \rightarrow J(z))$

g. [5 points] x did not see a junior at the park.
$\forall z\,(J(z) \to \neg S(x,z))$ (or $\forall z\,\big(S(x,z) \to \neg J(z)\big)$ or $\neg\exists z(J(z) \wedge S(x,z))$)

h. [5 points] A junior saw x at the park.
$\exists z\,(J(z) \wedge S(z,x))$

i. [5 points] A person who saw x at the park did not see y at the park.
$\exists z\,(S(z,x) \wedge \neg S(z,y))$

j. [5 points] If x saw y at the park, then there is a junior that saw x at the park.
$S(x,y) \to (\exists z\, J(z) \wedge S(z,x))$

k. [5 points] x is a junior who saw y at the park but there is another junior that also saw y at the park.
$J(x) \wedge S(x,y) \wedge \exists z(J(z) \wedge \neg(z = x) \wedge S(z,y))$

Problem 6. [10 points]
Rewrite each of the following so all negation symbols are directly in front predicates. Note that the logical equivalence laws (e.g., Commutative, Associative, etc.) for propositional logic also apply to predicate logic.

a. [5 points] $\neg\exists x(P(x) \wedge Q(x))$
$\forall x(\neg P(x) \vee \neg Q(x))$

b. [5 points] $\neg\forall x\exists y\forall z(R(x,y) \to Q(z,y))$
$\exists x\forall y\exists z(R(x,y) \wedge \neg Q(z,y))$