

1 0 1 1_{two}

$$\begin{aligned} & (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)_{\text{ten}} \\ = & (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)_{\text{ten}} \\ = & 8 + 0 + 2 + 1_{\text{ten}} \\ = & 11_{\text{ten}} \end{aligned}$$

Converting to base ten.

1) What is 1110_{two} in base ten?

Answer

14

$$\begin{aligned} & (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ & = 8 + 4 + 2 + 0 = 14 \end{aligned}$$

1) Of a doubleword's 64 bits, what is the leftmost bit numbered?

- 64
 63

Correct

The numbering of the rightmost bit starts with 0, so the leftmost bit is numbered with 63 rather than 64.

2) Given the following 64-bit number, what is the most significant bit's value?

1000 0000 0000 0000 0000 0000 0000 0000 0000 0000 0000 0000 0000 0000 0000 0000

- 1
 0

Correct

The leftmost bit is the most significant bit, meaning the bit with the greatest weight. Using the term "most significant bit" versus "leftmost bit" helps prevent confusion if a word is drawn vertically.

1) What is the largest base ten number representable in 4 bits (assuming the "natural" representation)?

- 8
- 15
- 16

Correct

1111_{two} is 15_{ten} . The number can also be computed as $2^4 - 1$.



2) What is the largest base ten number representable in 8 bits (assuming the "natural" representation)?

- 255
- 256

Correct

$2^8 - 1$ is 255. That largest number in base two is 11111111_{two} .



3) What is the largest base ten number approximately representable by 32 bits (assuming the "natural" representation)?

- 4 million
- 4 billion
- 4 trillion

Correct

$2^{32} - 1$ is 4,294,967,295, or just over 4 billion.



4) How is the largest base ten number representable by 64 bits calculated (assuming the "natural" representation)?

- $2^{63} - 1$
- 2^{64}
- $2^{64} - 1$

Correct

$2^{64} - 1$ is approximately 18.5 quintillion.



1) Sign and magnitude representation and two's complement representation are used about equally in modern computers.

- True
 False

Correct

All computers use two's complement. Sign and magnitude representation was tried in early computers, but was difficult to implement efficiently in hardware, and the existence of both a positive and negative zero was problematic for programmers.



2) In two's complement, is the following number positive or negative?
11110000 00000000 00000000 00000000 00000000 00000000
00000000 00000000

- Positive
 Negative

Correct

The leftmost bit is the sign bit. The 1 means negative.



3) In two's complement, is the following number positive or negative?
00000000 00000000 00000000 00000000 00000000 00000000
00000000 00001111

- Positive
 Negative

Correct

The leftmost bit is the sign bit. The 0 means positive.



4) Knowing that 2^{63} is 9,223,372,036,854,775,808, what is the base ten value of the following two's complement number?

10000000 00000000 00000000 00000000 00000000 00000000
00000000 00000000

- 9,223,372,036,854,775,808
 -1

Correct

The leftmost bit is multiplied by -2^{63} , then added with the remaining bits that are multiplied by those bits' usual positive base values. Because those remaining bits are all 0's, the base ten value is just $-9,223,372,036,854,775,808 + 0 = -9,223,372,036,854,775,808$.



5) How is 0 represented in two's complement?

All 0's: 00000000 00000000 00000000 00000000 00000000
00000000 00000000 00000000

or

All 1's: 11111111 11111111 11111111 11111111 11111111
11111111 11111111 11111111

- All 0's
 All 1's

Correct

$0 \times -2^{64} + 0 \times 2^{63} + 0 \times 2^{62} \dots = 0$.

All 1's actually represents -1: The first 1 is multiplied by -2^{63} , and the remaining 63 1's summed yields $2^{63} - 1$, so the sum is -1.



6) In a two's complement representation, the magnitude of the largest negative value is one greater than the magnitude of the largest positive number.

- True
 False

Correct

Zero is one of the positive values, leaving one less value available for the other positives. Ex: For an 8-bit two's complement representation, the most negative value is -128 (10000000), while the most positive value is 127 (01111111).



Indicate if the binary operation resulted in overflow. The numbers presented are 32-bit values; 64-bit values do not fit in the space, but the concepts are identical no matter the number of bits.

1)

```

1000 1111 0000 0000 0000 0000 0000 0000
+ 1000 0000 1111 1111 1111 1111 0000 0000
-----
0000 1111 1111 1111 1111 1111 0000 0000
    
```

- Overflow
 No overflow

Correct

Overflow occurs when the numbers' sign bits match, but yield a sum with a different sign bit.

The leftmost bits (sign bits) are added, which results in 0 with a carry of 1. A negative result is expected, but because numbers are represented as 32-bit values the carry bit is lost and the result appears positive. Ex: $(-1,895,825,408) + (-2,130,706,688) \neq 268,435,200$.



2)

```

0000 1111 0000 0000 0000 0000 0000 0000
+ 0111 0000 0000 0000 0000 0000 0000 0000
-----
0111 1111 0000 0000 0000 0000 0000 0000
    
```

- Overflow
 No overflow

Correct

The numbers being added are positive, and the result is positive (as indicated by the 1 in the leftmost bit). No overflow has occurred. Ex: $251,658,240 + 1,879,048,192 = 2,130,706,432$.



3)

```

0111 0000 0000 0000 0000 0000 0000 0000
+ 1111 0000 0000 0000 0000 0000 0000 0000
-----
0110 0000 0000 0000 0000 0000 0000 0000
    
```

- Overflow
 No overflow

Correct

A positive number is being added to a negative number of a smaller magnitude, therefore overflow does not occur. Ex: $1,879,048,192 + (-268,435,456) = 1,610,612,736$.



1) If $+3_{10}$ is 00000011_{2} , what is -3_{10} in an 8-bit two's complement representation?

Check Show answer

Answer

11111101

After inverting: 11111100
 After adding 1: 11111101
 Thus 11111101_{2} is -3_{10} .



2) What is -9_{10} in an 8-bit two's complement representation?

Check Show answer

Answer

11110111

$+9$ is 00001001 .
 After inverting: 11110110
 After adding 1: 11110111
 Thus, 11110111_{2} is -9_{10} .



3) Assuming a 64-bit two's-complement representation, what is $11111111 11111111 11111111 11111111 11111111 11111111 11111111 11111100_{2}$ in base ten?

Check Show answer

Answer

-4

After inverting: $00000000 \dots 00000011$
 After adding 1: $00000000 \dots 00000100$
 In base ten: -4



1) What is 0011 as a hexadecimal digit?

Check

Show answer

2) What is 1011 as a hexadecimal digit?

Check

Show answer

3) What is 11110000 in 2-digit hexadecimal? Write answer as: a1

Check

Show answer

4) What is 2f in 8-bit binary?

Check

Show answer

Answer

3

Hexadecimal has symbols 0 - 9, so 0011 is just 3. Beyond 9, hexadecimal uses letters a - f.

Answer

b

Hexadecimal uses symbols a - f for binary values 1010 - 1111 (decimal values 10 - 15). a is 1010, b is 1011, etc.

Answer

f0

1111 is f.
0000 is 0.
Together 11110000 is f0.
Hexadecimal allows representing bits using fewer digits than binary.

Answer

00101111

2 is 0010.
f is 1111.
Together the bits are 2f.
Hexadecimal allows representing bits using fewer digits than binary.