Homework Assignment 6
CS 2233
Sections 001 and 002
Due: 11:59pm Friday, March 22

Problem 1. [10 points]
Complete all participation activities in zyBook sections 8.1-8.5

Problem 2. [10 points]
What are the first 4 terms of the following sequences:

a. [5 points] $\{(-3)^n\}_{n \in \boldsymbol{N}}$

b. [5 points] $\{(-1)^n + 1\}_{n \in \boldsymbol{N}}$

Problem 3. [10 points]
Use index substitution to rewrite the following summation so that the index starts at 0. Then use the closed form of the geometric series to compute the value of the summation:

$$\sum_{i=3}^{6} (-2)^{i-3}$$

Problem 4. [15 points]
Express each sequence below using compact notation.

a. [5 points] $4, 10, 16, 22, 28, 34, 40, \ldots$

b. [5 points] $5, 15, 45, 135, 405, \ldots$

c. [5 points] $10, 20, 10, 20, 10, 20, 10, \ldots$

Problem 5. [20 points] Consider a proof by mathematical induction on the positive integers of $P(n)$ where $P(n)$ is: The sum of the first n even positive integers is $n^2 + n$.

a. [5 points] What is the statement $P(1)$?

b. [5 points] What is the induction hypothesis?

c. [5 points] What do you need to prove in the inductive step?

d. [5 points] Complete the proof of the inductive step.

Problem 6. [5 points] Use mathematical induction to prove $3^n < n!$ for all integers $n > 6$.

Problem 7. [5 points] Use mathematical induction to prove $\log(n!) \leq n\log(n)$ for all integers $n \geq 1$.

Note that:
1) $\log(1) = 0$
2) $\log(a \cdot b) = \log(a) + \log(b)$
3) If $a \leq b$ then $\log(a) \leq \log(b)$

Problem 8. [5 points] Prove by mathematical induction on the positive integers $\forall n P(n)$ where $P(n)$ is: If $A_1, A_2, \cdots, A_n$ and B are sets, then $(\bigcap_{i=1}^{n} A_i) \cup B = \bigcap_{i=1}^{n}(A_i \cup B)$.

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Note:

1. $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$
2. $\bigcap_{i=1}^{n} (A_i \cup B) = (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B)$

Hints:

1. For each $k \geq 1$, $\bigcap_{i=1}^{k+1} A_i = \left(\bigcap_{i=1}^{k} A_i\right) \cap A_{k+1}$
2. When X, Y, and Z are sets, $(X \cap Y) \cup Z = (X \cup Z) \cap (Y \cup Z)$