# Section 6.2
# Properties of Binary Relations

# Properties of Relations: Reflexivity

- A relation $R$ on a set $A$ is reflexive if $(a, a) \in R$ for each $a \in A$
  - I.e., $aRa$ for each $a \in A$

# Properties of Relations: Reflexivity

- Example: Let $A = \{1, 2, 3, 4\}$. The following relations on $A$ are reflexive:
  - $R_0 = \{(1,1), (2,2), (3,3), (4,4)\}$
  - $R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$
  - $R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

# Properties of Relations: Antireflexivity

- A relation $R$ on a set $A$ is <u>antireflexive</u> if $(a, a) \notin R$ for each $a \in A$

# Properties of Relations: Antireflexivity

- Example: Let $A = \{1, 2, 3, 4\}$. The following relations on $A$ are antireflexive:
  - $\{ \}$
  - $\{(1,2), (1,4), (2,1), (4,1) \}$
  - $\{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4) \}$

# Properties of Relations: Antireflexivity

- Example: Let $A = \{1, 2, 3, 4\}$. The following relations on $A$ are neither reflexive nor antireflexive:
  - $\{(1,1)\}$
  - $\{(1,2), (1,4), (2,1), (4,1)\ , (4, 4)\}$
  - $\{(1,2), (1,3), (1,4), (2, 2), (2,3), (2,4), (3, 3), (3,4)\ \}$
  - $\{(1,1), (2,2), (3,3)\}$

# Properties of Relations: Symmetry and Antisymmetry

- A relation $R$ on a set $A$ is symmetric if $(a, b) \in R$ whenever $(b, a) \in R$ for all $a, b \in A$
- A relation $R$ on a set $A$ is antisymmetric if for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$ then $a = b$
  - $R$ is antisymmetric if there are no distinct elements $a$ and $b$ such that $(a, b) \in R$ and $(b, a) \in R$
  - In general, If R is antisymmetric, then it can be either reflexive or not reflexive

# Properties of Relations: Symmetry and Antisymmetry

- Example: Let $A = \{1, 2, 3, 4\}$. The following relations on $A$ are symmetric:
  - $R_2 = \{(1,1), (1,2), (2,1)\}$
  - $R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$
  - Note that $R_2$ is not reflexive and $R_3$ is reflexive

# Properties of Relations: Symmetry and Antisymmetry

- Example: Let $A = \{1, 2, 3, 4\}$. The following relations on $A$ are antisymmetric:
  - $R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$
  - $R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$
  - $R_6 = \{(3,4)\}$
  - Note that $R_4$ and $R_6$ are not reflexive and $R_5$ is reflexive

# Properties of Relations: Symmetry and Antisymmetry

- Example: Let $A = \{1, 2, 3, 4\}$. The following relations on $A$ are both symmetric and antisymmetric:
  - $\{\ \}$
  - $\{(1,1)\}$
  - $\{(1,1), (2, 2)\}$
  - $\{(2, 2), (3, 3), (4, 4)\}$

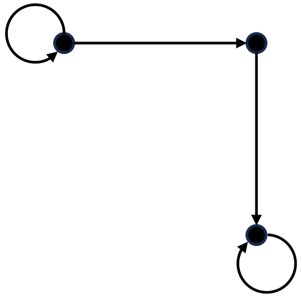
# Properties of Relations: Transitivity

- A relation $R$ on a set $A$ is transitive if whenever $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ for all $a, b, c \in A$

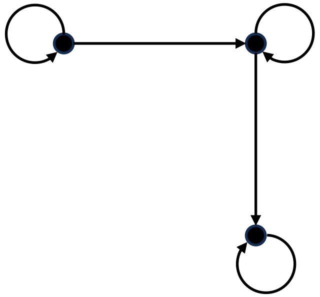
# Properties of Relations: Transitivity

- Example 13: Let $A = \{1, 2, 3, 4\}$. The following relations on $A$ are transitive:
  - $R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$
  - $R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$
  - $R_6 = \{(3,4)\}$

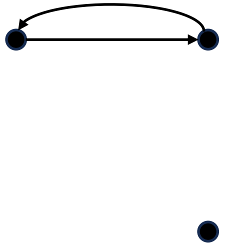
# More Examples



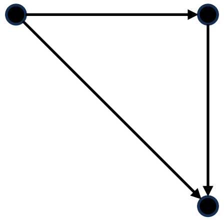
Not reflexive
Not symmetric
Not transitive



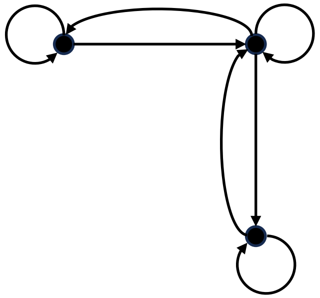
Reflexive
Not symmetric
Not transitive



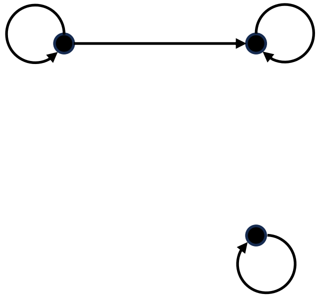
Not reflexive
Symmetric
Not transitive



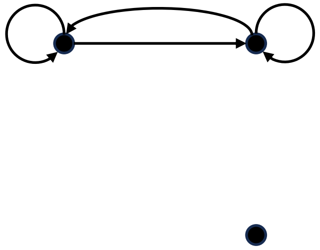
Not reflexive
Not symmetric
Transitive



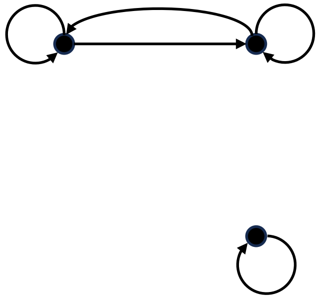
Reflexive
Symmetric
Not transitive



Reflexive
Not symmetric
Transitive



Not reflexive
Symmetric
Transitive



Reflexive
Symmetric
Transitive