CS 3333: Mathematical Foundations

Number Systems

Number Systems

- The **Decimal Number System** is the system that we generally use to express integers.

Number Systems

- The **Decimal Number System** is the system that we generally use to express integers.
- The number 754 can be written as $7 \cdot 100 + 5 \cdot 10 + 4 \cdot 1$

Number Systems

- The **Decimal Number System** is the system that we generally use to express integers.
- The number 754 can be written as $7 \cdot 100 + 5 \cdot 10 + 4 \cdot 1 = 7 \cdot 10^2 + 5 \cdot 10^1 + 4 \cdot 10^0$.

Number Systems

- The **Decimal Number System** is the system that we generally use to express integers.
- The number 754 can be written as $7 \cdot 100 + 5 \cdot 10 + 4 \cdot 1 = 7 \cdot 10^2 + 5 \cdot 10^1 + 4 \cdot 10^0$.
- Radix: 10, Symbols: $\{0, 1, \ldots, 9\}$, Position weight: 10^x.

Number Systems

- This applies to real numbers as well.

Number Systems

- This applies to real numbers as well.
- 12.56

Number Systems

- This applies to real numbers as well.
- $12.56 = 1 \cdot 10^1 + 2 \cdot 10^0 + 5 \cdot 10^{-1} + 6 \cdot 10^{-2}$.

Number Systems

- **Theorem 1**: Let b be an integer greater than 1. Then if n is a positive integer, it can be expressed uniquely in the form $n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_1 b + a_0$ where k is a nonnegative integer, $a_0, a_1, \ldots, a_k$ are nonnegative integers less than b, and $a_k \neq 0$.

Number Systems

- **Theorem 1**: Let b be an integer greater than 1. Then if n is a positive integer, it can be expressed uniquely in the form $n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_1 b + a_0$ where k is a nonnegative integer, $a_0, a_1, \ldots, a_k$ are nonnegative integers less than b, and $a_k \neq 0$.
- The number is written $(a_k a_{k-1} \ldots a_1 a_0)_b$ (subscript generally omitted when $b = 10$).

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.
- The symbols used are the nonnegative integers less than 2 ($\{0, 1\}$).

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.
- The symbols used are the nonnegative integers less than 2 $(\{0, 1\})$.
- Example: 110_2

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.
- The symbols used are the nonnegative integers less than 2 ($\{0, 1\}$).
- Example: $110_2 = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.
- The symbols used are the nonnegative integers less than 2 ($\{0, 1\}$).
- Example: $110_2 = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 4 + 2 = 6$.

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.
- The symbols used are the nonnegative integers less than 2 ($\{0, 1\}$).
- Example: $110_2 = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 4 + 2 = 6$.
- Example: 101.11_2

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.
- The symbols used are the nonnegative integers less than 2 ($\{0, 1\}$).
- Example: $110_2 = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 4 + 2 = 6$.
- Example: $101.11_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2}$

Number Systems

- A number system that commonly occurs in computer science is the **binary number system** which is the number system with radix $= 2$.
- The symbols used are the nonnegative integers less than 2 ($\{0, 1\}$).
- Example: $110_2 = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 4 + 2 = 6$.
- Example: $101.11_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2} = 4 + 1 + 1/2 + 1/4 = 5.75$.

Number Systems

- Another number system is the **octal number system**.

Number Systems

- Another number system is the **octal number system**.
- Radix $= 8$, symbols $= \{0, 1, \ldots, 7\}$.

Number Systems

- Another number system is the **octal number system**.
- Radix $= 8$, symbols $= \{0, 1, \ldots, 7\}$.
- Example: 120_8

Number Systems

- Another number system is the **octal number system**.
- Radix $= 8$, symbols $= \{0, 1, \ldots, 7\}$.
- Example: $120_8 = 1 \cdot 8^2 + 2 \cdot 8^1 + 0 \cdot 8^0$

Number Systems

- Another number system is the **octal number system**.
- Radix $= 8$, symbols $= \{0, 1, \ldots, 7\}$.
- Example: $120_8 = 1 \cdot 8^2 + 2 \cdot 8^1 + 0 \cdot 8^0 = 64 + 16 = 80$.

Number Systems

- Yet another number system is the **Hexadecimal number system**.

Number Systems

- Yet another number system is the **Hexadecimal number system**.
- Radix $= 16$, symbols $= \{0, 1, \ldots, 9, A, B, C, D, E, F\}$.

Number Systems

- Yet another number system is the **Hexadecimal number system**.
- Radix $= 16$, symbols $= \{0, 1, \ldots, 9, A, B, C, D, E, F\}$.
- Example: 120_{16}

Number Systems

- Yet another number system is the **Hexadecimal number system**.
- Radix $= 16$, symbols $= \{0, 1, \ldots, 9, A, B, C, D, E, F\}$.
- Example: $120_{16} = 1 \cdot 16^2 + 2 \cdot 16 + 0 \cdot 16^0$

Number Systems

- Yet another number system is the **Hexadecimal number system**.
- Radix $= 16$, symbols $= \{0, 1, \ldots, 9, A, B, C, D, E, F\}$.
- Example: $120_{16} = 1 \cdot 16^2 + 2 \cdot 16 + 0 \cdot 16^0 = 256 + 32 = 288$.

Number Systems

- How can we convert a decimal number to binary?
- Example: What is 215_{10} in binary?

Number Systems

- How to convert a base-b representation of a decimal number:

Number Systems

- How to convert a base-b representation of a decimal number:
  1. Let the given integer be denoted N.

Number Systems

- How to convert a base-b representation of a decimal number:
  1. Let the given integer be denoted N.
  2. By the division algorithm, there exist unique integers N_1 and $0 \leq a_0 < b$ such that $N = N_1 \cdot b + a_0$. a_0 is the rightmost digit of the base-b representation of N.

Number Systems

- How to convert a base-b representation of a decimal number:
  1. Let the given integer be denoted N.
  2. By the division algorithm, there exist unique integers N_1 and $0 \leq a_0 < b$ such that $N = N_1 \cdot b + a_0$. a_0 is the rightmost digit of the base-b representation of N.
  3. Apply the division algorithm to N_1 to get $N_1 = N_2 \cdot b + a_1$. a_1 is the next digit of the base-b representation of N.

Number Systems

- How to convert a base-b representation of a decimal number:
  1. Let the given integer be denoted N.
  2. By the division algorithm, there exist unique integers N_1 and $0 \leq a_0 < b$ such that $N = N_1 \cdot b + a_0$. a_0 is the rightmost digit of the base-b representation of N.
  3. Apply the division algorithm to N_1 to get $N_1 = N_2 \cdot b + a_1$. a_1 is the next digit of the base-b representation of N.
  4. Repeat this until the quotient $N_k = 0$.

Number Systems

- How to convert a decimal fraction to base-b representation:

Number Systems

- How to convert a decimal fraction to base-b representation:
    1. Multiply $0.f$ with b. Let the resulting number be $x.y$ where x is the integer part and y is the new fractional part.

Number Systems

- How to convert a decimal fraction to base-b representation:
    1. Multiply $0.f$ with b. Let the resulting number be $x.y$ where x is the integer part and y is the new fractional part.
    2. Use x as the next fractional digit in the base-b representation.

Number Systems

- How to convert a decimal fraction to base-b representation:
  1. Multiply $0.f$ with b. Let the resulting number be $x.y$ where x is the integer part and y is the new fractional part.
  2. Use x as the next fractional digit in the base-b representation.
  3. If $y \neq 0$, then repeat procedure with $0.y$.

Number Systems

- **Converting between Binary and Octal**:

Number Systems

- **Converting between Binary and Octal**:
-

Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

Number Systems

- **Converting between Binary and Octal**:
Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111
- Binary $\rightarrow$ Octal:

Number Systems

- **Converting between Binary and Octal**:
-
Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

- Binary $\rightarrow$ Octal:
  - Group binary bits into groups of 3 and use their decimal value as an octal digit.

Number Systems

- **Converting between Binary and Octal**:
-
Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

- Binary $\rightarrow$ Octal:
  - Group binary bits into groups of 3 and use their decimal value as an octal digit.
  - Can pad the left end of integer part (and right of fractional part) with 0s as needed to form groups of 3.

Number Systems

- **Converting between Binary and Octal**:
Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111
- Binary $\rightarrow$ Octal:
  - Group binary bits into groups of 3 and use their decimal value as an octal digit.
  - Can pad the left end of integer part (and right of fractional part) with 0s as needed to form groups of 3.
- Octal $\rightarrow$ Binary:

Number Systems

- **Converting between Binary and Octal**:
-

Octal	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

- Binary $\rightarrow$ Octal:
  - Group binary bits into groups of 3 and use their decimal value as an octal digit.
  - Can pad the left end of integer part (and right of fractional part) with 0s as needed to form groups of 3.
- Octal $\rightarrow$ Binary:
  - Expand each digit into the equivalent binary code.

Number Systems

- **Converting between Binary and Hexadecimal**

Number Systems

- **Converting between Binary and Hexadecimal**
- Similar to Octal, except we use 4 bits for each hexadecimal symbol.

Number Systems

- **Converting between Binary and Hexadecimal**
- Similar to Octal, except we use 4 bits for each hexadecimal symbol.
-

Dec	Binary	Hex		Dec	Binary	Hex
0	0000	0		8	1000	8
1	0001	1		9	1001	9
2	0010	2		10	1010	A
3	0011	3		11	1011	B
4	0100	4		12	1100	C
5	0101	5		13	1101	D
6	0110	6		14	1110	E
7	0111	7		15	1111	F