Pigeonhole Principle: If k is a positive integer and $k+1$ or more objects are to be placed in k boxes, then at least one box contains at least two objects.

- For example, 3 boxes and 4 objects. Then, $k = 3$.
- We can place objects in boxes randomly, but when we place object $k+1$, it must go in a box that already had an object.
- In other words, there must be at least one box contains at least two objects.

Case 1:	1	1	2
Case 2:	1	3	0
Case 3:	2	2	0
Case 4:	3	1	0
Case 5:	4	0	0
...			

[KR 6.2] Example 3: The possible scores on a test are $0, 1, \ldots 100$. What is the minimum number of students that must take the test to guarantee that at least two students have the same score?

- 101 different grades - "Boxes"
- $k = 101$
- if $k + 1 = 102$ students take the exam, at least two of them have the same score.

[KR 6.2] Exercise 3a: A drawer has 12 brown socks and 12 black socks, randomly placed. If you draw socks at random without looking into the drawer, how many socks do you need to draw to guarantee you have a pair of socks of the same color?

- Boxes are colors, $k = 2$
- So, if we draw $k + 1 = 3$ socks, we will have a matching pair

Exercise 3b: How many do you have to take out in order to guarantee you have at least two brown socks?

- In the worst case, we draw all 12 black socks first.
- Then, we need to draw two more brown socks.
- The total is 14.

Exercise 7: Show that for any group of 5 integers (not necessarily consecutive), at least two of them have the same remainder when divided by 4.

- If a is an integer, then $a\%4$ is one of 4 outcomes: 0, 1, 2, 3
- These outcomes are our boxes, $k = 4$.
- Since we have $k + 1$ integers, at least two of them, must go in the same box.

Generalized Pigeonhole Principle: If $N \geq 0$ objects are placed in $k \geq 1$ boxes, then at least one of the boxes has at least $\lceil N/k \rceil$ objects.

Example 5: In any group of 100 people, at least $\lceil 100/12 \rceil = 9$ are born in the same month.

- There are 12 months or boxes.
- $\frac{100}{12} = \lceil 8.33 \rceil = 9$

[KR 6.2] Example 6: What is the minimum number of students that must take a test to ensure that at least 3 students receive the same grade (where a grade can be A, B, C, D, or F)?

- $k = 5$ and $\lceil \frac{N}{k} \rceil = 3$
- What is the minimum N, such that $\lceil \frac{N}{5} \rceil = 3$?
- We need the smallest N, such that $\frac{N}{5} > 2$
- $\frac{10}{5} = 2$. So, $N = 11$.

Problem 15a: Show that if 5 integers are selected from the first eight positive integers (1 through 8), then there must be a pair of selected integers whose sum is 9.

Numbers that sum to 9:

1 2 3 4 5 6 7 8

Boxes: $\boxed{}_{1,8}$ $\boxed{}_{2,7}$ $\boxed{}_{3,6}$ $\boxed{}_{4,5}$

- After drawing 5 numbers, one box must have two numbers in it. We do not draw a number twice, so we must have draw both number associated with the box.

Problem 21: Suppose that in a discrete math class we have 25 students and each of them are either a freshman, sophomore, or junior.

1. Show that there are either at least 9 freshmen, 9 sophomores, or 9 juniors.

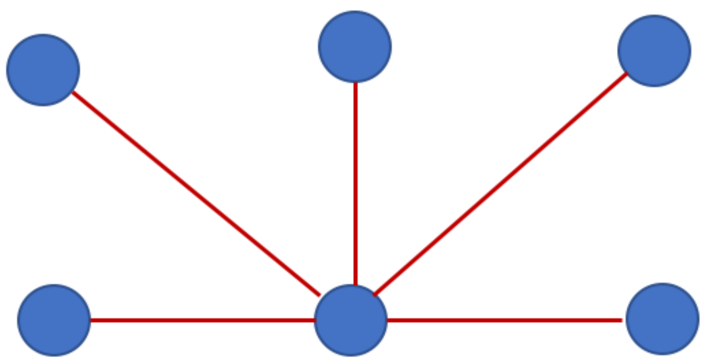
   - We will prove that the negation is not possible.
   - Negation: ≤ 8 freshmen, ≤ 8 sophomores, and ≤ 8 juniors.
   - If negation is true, then max number of students in the class is $8 + 8 + 8 = 24$. But our class has 25 students.

2. Show that there are either at least 3 freshmen, at least 19 sophomores, or at least 5 juniors in the class.

   - The same idea.
   - $\leq$ 2 freshmen, $\leq$ 18 sophomores, $\leq$ 4 juniors would imply $\leq$ 24 students, but our class has 25 students.

KR 6.2 Problem 39: Consider a network of 6 computers. Each computer is directly connected to 0 or more of the other computers. Show that there are at least 2 computers that have the same number of direct connections.

- Boxes are connections or boxes. Possibilities: 0, 1, 2, 3, 4, 5
- Pigeonhole principle doesn't apply directly because the number of computers is same as the number of boxes.
- Only way it isn't true is if each box has 1 computer, but if there is a computer with 5 connections, then there are no computers with 0 connections.



Problem 46: Suppose there are 51 houses on a street whose addresses range from 1000 to 1099. Show that there are at least two houses which have addresses that are consecutive integers.

- If it is false, we could have all 51 houses with no consecutive addresses.
- If the first house takes 1000, the next cannot use 1001. It takes 1002. The third takes 1004. The fourth takes 1006, ...
- The 50th house takes 1098.
- the 51st house must take a number next to a selected number.

Example 7: Consider a standard deck of 52 cards.

1. How many cards must be drawn to ensure that at least 3 of these cards are from the same suit (diamonds, clubs, hearts and spades)?
   - First 8 cards could be 2 cards of each suit. 9th card will give 3 of some suit.
   - $\lceil \frac{N}{4} \rceil = 3$

2. How many cards must be drawn at random to guarantee at least 3 hearts are selected?
   - We may draw all cards from the other suits.
   - 13 spades + 13 clubs + 13 diamonds + 3 hearts = 42