CS 3333: Mathematical Foundations

Matrices

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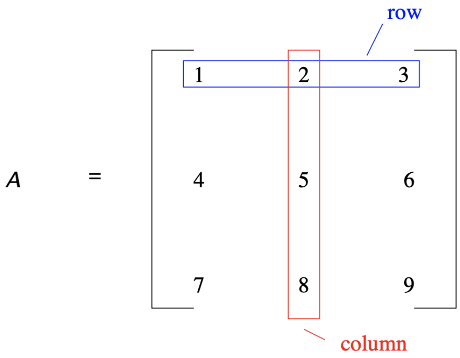

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- Note that a 2×4 matrix is not the same as a 4×2 matrix.

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  - Example: $\begin{pmatrix} 7 \\ -2 \\ 5 \\ 11 \end{pmatrix}$

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  - In other words, if a matrix is $n \times n$ for some integer $n > 0$, then it is a square matrix.

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- The main diagonal is defined for both square and non-square matrices. However it is more interesting and more commonly used in the case of square matrices.

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- Example: $I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

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- Example: $D_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}$
- Example: $D_3 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

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- A lower triangular matrix of order $n \times n$ is denoted L_n.
  - Example: $L_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 1 & 4 & 6 & 3 & 5 \end{pmatrix}$

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- An **upper triangular matrix** is a square matrix that may only have nonzero entries on the main diagonal and above the main diagonal.
- An upper triangular matrix of order $n \times n$ is denoted U_n.
  - Example: $U_5 = \begin{pmatrix} 2 & 0 & 1 & 2 & 3 \\ 0 & -7 & 0 & 1 & 1 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

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  - Example: $\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.7 & 0.3 & 0 \\ 0 & 0.5 & 0.5 \end{pmatrix}$

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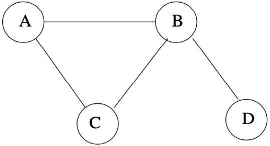

	A	B	C	D
A	1	1	1	0
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  - $3x_1 + 4x_2 = 7$
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  - $\begin{pmatrix} 3 & 4 & 0 \\ -2 & 0 & 7 \\ 2 & 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 20 \end{pmatrix}$

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- $\begin{pmatrix} 4 & 2 & 3 \\ 5 & 7 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 8 & 9 \\ 3 & 5 & 4 \end{pmatrix} = \begin{pmatrix} 4+1 & 2+8 & 3+9 \\ 5+3 & 7+5 & 6+4 \end{pmatrix} = \begin{pmatrix} 5 & 10 & 12 \\ 8 & 12 & 10 \end{pmatrix}$.

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- $\begin{pmatrix} 4 & 2 & 3 \\ 5 & 7 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 8 & 9 & 2 \\ 3 & 5 & 4 & 9 \end{pmatrix}$ is not defined.

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- $4 \cdot \begin{pmatrix} 3 & 2 & 5 \\ 6 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 4 \cdot 3 & 4 \cdot 2 & 4 \cdot 5 \\ 4 \cdot 6 & 4 \cdot 1 & 4 \cdot 7 \end{pmatrix} = \begin{pmatrix} 12 & 8 & 20 \\ 24 & 4 & 28 \end{pmatrix}$

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- $A = \begin{pmatrix} a_1 a_2 \cdots a_n \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$
- $A \cdot B = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$

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- An example of dot product
- $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$

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- $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$
- $A \cdot B = 1 * 4 + 2 * 5 + 3 * 6 = 4 + 10 + 18 = 32$

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- Suppose A is an $m \times p$ matrix and B is a $p \times n$ matrix (note the number of *columns* of A is the same as the number of *rows* of B). Then the **matrix multiplication** $A \cdot B$ is defined.

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  - Multiplication is defined because the number of columns of A is the same as the number of rows of B.
  - Result will be a 2×2 matrix.

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- The result is an $m \times n$ matrix (resulting matrix has the same number of rows as A and the same number of columns as B).
- The (i, j)th entry of the resulting matrix is the dot product of row i of A and column j of B.
- Example:
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    - Multiplication is defined because the number of columns of A is the same as the number of rows of B.
    - Result will be a 2×2 matrix.
    - Entry in position $(2, 1)$ in the resulting matrix will be the dot product of the 2nd row in A with the 1st column of B: $a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} + a_{24}b_{41}$.

Example of Matrix Multiplication

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- $A * B$ is not defined since A's columns does not equal to B's rows.
- However, $B * A$ is defined and its result is 3×2

Example of Matrix Multiplication

- $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$
- $A * B$ is not defined since A's columns does not equal to B's rows.
- However, $B * A$ is defined and its result is 3×2
- $B * A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} * \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \\ 23 & 34 \end{pmatrix}$
- $C_{11} = \begin{pmatrix} 1 & 2 \end{pmatrix} * \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 1 * 1 + 2 * 3 = 7$
- $C_{12} = \begin{pmatrix} 1 & 2 \end{pmatrix} * \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 1 * 2 + 2 * 4 = 10$
- $C_{21} = 3 * 1 + 4 * 3 = 15,\ C_{22} = 3 * 2 + 4 * 4 = 22$
- $C_{31} = 5 * 1 + 6 * 3 = 23,\ C_{32} = 5 * 2 + 6 * 4 = 34$