CS 3333: Mathematical Foundations

Eigenvalues and Eigenvectors

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  - $(AB)^{-1} = B^{-1} \cdot A^{-1}$ if A, B are non-singular $n \times n$ matrices

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- Let B be a matrix after swapping two rows of A. Then $|A| = -|B|$.
  - $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $|A| = 1 * 4 - 3 * 2 = -2$
  - Get matrix B after swapping row 1 and row 2 in A.
  - $B = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$, $|B| = 3 * 2 - 1 * 4 = 2$

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Properties of the determinant of matrices after applying elementary row operations:

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  - Get matrix B after multiplying row 1 of A by 2.
  - $B = \begin{pmatrix} 2 & 4 \\ 3 & 4 \end{pmatrix}$, $|B| = 2 * 4 - 4 * 3 = -4$

Eigenvalues and Eigenvectors

Properties of the determinant of matrices after applying elementary row operations:

- Let B be a matrix after multiplying some row of A by a scalar and then adding it onto another row of A. Then $|A| = |B|$.
  - $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $|A| = 1 * 4 - 3 * 2 = -2$
  - Get matrix B after multiplying row 1 of A by -3 and then adding it onto row 2 of A.
  - $B = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$, $|B| = 1 * (-2) - 0 * 2 = -2$

Eigenvalues and Eigenvectors

- Consider an equation of the form $A \cdot x = \lambda \cdot x$ where A is an $n \times n$ matrix of knowns, x is an $n \times 1$ vector of unknowns, and λ is an unknown scalar.

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- If the equation is satisfied for some vector x where x is not a null vector, then x is an **eigenvector** and λ is an **eigenvalue**.

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- $|A - \lambda \cdot I| = 0$ is called the **characteristic equation of** A.

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- $\lambda \cdot I = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$
- Then $A - \lambda \cdot I = \begin{pmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ a_{21} & a_{22} - \lambda & a_{23} \\ a_{31} & a_{32} & a_{33} - \lambda \end{pmatrix}$.

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- Need to find λ such that $\begin{vmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ a_{21} & a_{22} - \lambda & a_{23} \\ a_{31} & a_{32} & a_{33} - \lambda \end{vmatrix} = 0$.

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- So, $(4-\lambda)(2-\lambda) - 3 = 0$
- Then, $\lambda = 1,\ 5$.

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- When $\lambda = 1$,
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- $A \cdot x = \lambda \cdot x \rightarrow \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 5 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

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- $$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ -1 & 2 & 3 \end{pmatrix}$$

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- The eigenvalues are $\lambda = 0$ and $\lambda = 2$.
- When $\lambda = 0$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
- When $\lambda = 2$, $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

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- The sum of the eigenvalues of A is equal to $\mathrm{tr}(A)$.
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  - $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$, $tr(A) = 4 + 2 = 6$, $\lambda_1 + \lambda_2 = 1 + 5 = 6$
- The product of the eigenvalues of A is equal to $|A|$.
  - $|A| = 4 * 2 - 3 * 1 = 5$, $\lambda_1 \cdot \lambda_2 = 1 \cdot 5 = 5$