Chapter 1
Logic

Logic

- A logic consists of:
  - A language for describing aspects of some worlds (syntax)
  - Rules for determining the meaning of sentences in the language (semantics)
  - Rules for deriving true sentences from other true sentences

- Logic is useful because:
  - It has no ambiguity
  - It can be used to find new true statements

Logic in Programming

- You have already been exposed to logic in programming
  - `((x >= 1) && (x <= 10))`
  - But no rules for deriving true sentences

Propositional Logic

- A proposition is a statement that is either true or false
- Examples
  - San Antonio is a city in Texas
  - 1 + 1 = 3
- Commands and questions are not propositions
- Noun phrases such as: "The store where I bought a pen yesterday" are not propositions
- Propositional formulas are built from atomic propositions and logical operators

Atomic Propositions

- Atomic propositions are the simplest type of proposition and are represented by propositional variables (usually named $p, q, r, s, \ldots$)
- A propositional variable can have one of two values: true (represented by T) or false (represented by F) indicating the truth or falsity of the propositions that it represents

Compound Propositions

- A compound proposition is created by using a logical operator (connective) and one or two other propositions
- The meaning of each operator can be described by a truth table

Compound Propositions - Negation

- $\neg p$
- Truth table:

p	$\neg p$
T	F
F	T

- "It is not the case that p", "not p"

Compound Propositions - Negation

- Assume p represents "Vandana’s smartphone has at least 32 GB of memory"
- Then $\neg p$ represents
  - "It is not the case that Vandana’s smartphone has at least 32 GB of memory"
  - or "Vandana’s smartphone has less than 32 GB of memory"

Compound Propositions - Conjunction

- $p \wedge q$
- Truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- "p and q"

Compound Propositions - Conjunction

- Assume p represents “Rebecca’s PC has more than 16 GB free hard disk space”
- and q represents “The processor in Rebecca’s PC runs faster than 1 GHz.”
- Then $p \wedge q$ represents
  - “Rebecca’s PC has more than 16 GB free hard disk space, and the processor in Rebecca’s PC runs faster than 1 GHz.”

Compound Propositions - Conjunction

- We use "but" and "even though" and other phrases as a conjunction often to indicate a contrast
  - I walked 30 miles, but my feet are not sore
  - My feet are not sore even though I walked 30 miles
  - I walked 30 miles, and despite that, my feet are not sore

Compound Propositions - Disjunction

- $p \vee q$
- Truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- "p or q"

Compound Propositions - Disjunction

- Example
- Assume p represents "The dog is sleeping"
- and q represents "The cat is sleeping"
- Then $p \vee q$ represents
  - "The dog is sleeping or the cat is sleeping"

Compound Propositions - Disjunction

- Be careful when translating from English to propositional logic

"You may have cake for dessert or you may have pie for dessert"

- Let p represent "You may have cake for dessert"
- Let q represent "You may have pie for dessert"
- This might be translated as $p \vee q$
- But it might better be translated as $(p \vee q) \wedge \neg(p \wedge q)$

Compound Propositions – Exclusive Or

- $p \oplus q$
- Truth table:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- "p or q but not both", "p x-or q"

Compound Propositions – Exclusive Or

- Assume p represents “The fastest runner is Sandy”
- and q represents “The fastest runner is Chris”
- Then $p \oplus q$ represents
  - "The fastest runner is Sandy, or the fastest runner is Chris, but not both”

Truth Tables for Compound Propositions

- We can determine the truth of any compound proposition in a truth table that uses the variables in the proposition
- The rows of the table correspond to possible combinations of truth values (true and false) for the variables.
  - If the proposition has one variable, then the table has 2 rows
  - If the proposition has two variables, then the table has 4 rows
  - If the proposition has n variables, then the table has 2^n rows
- There is a column for each sub-proposition and for the entire proposition (one column for each variable and operator)

Truth Tables for Compound Propositions

- Example: $(p \wedge \neg q) \vee q$
- Create a table with 4 rows and with columns for the proposition and all sub-propositions

p	q	$\neg q$	$(p \wedge \neg q)$	$(p \wedge \neg q) \vee q$

Truth Tables for Compound Propositions

- Example: $(p \wedge \neg q) \vee q$
- Fill out the p and q columns with all possible combinations of truth values

p	q	$\neg q$	$(p \wedge \neg q)$	$(p \wedge \neg q) \vee q$
T	T			
T	F			
F	T			
F	F			

Truth Tables for Compound Propositions

- Example: $(p \land \neg q) \lor q$
- Complete the $\neg q$ column using the q column

p	q	$\neg q$	$(p \land \neg q)$	$(p \land \neg q) \lor q$
T	T	F		
T	F	T		
F	T	F		
F	F	T		

Truth Tables for Compound Propositions

- Example: $(p \land \neg q) \lor q$
- Complete the $(p \land \neg q)$ column using the p and $\neg q$ columns

p	q	$\neg q$	$(p \land \neg q)$	$(p \land \neg q) \lor q$
T	T	F	F	
T	F	T	T	
F	T	F	F	
F	F	T	F	

Truth Tables for Compound Propositions

- Example: $(p \wedge \neg q) \vee q$
- Complete the $(p \wedge \neg q) \vee q$ column using the $(p \wedge \neg q)$ and q columns

p	q	$\neg q$	$(p \wedge \neg q)$	$(p \wedge \neg q) \vee q$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	F

Example

- Let p represent "Chris went to the store"
- Let q represent "Sandy went to the store"

- Translate the following into propositional logic:
  1. It is not the case that both Chris went to the store and Sandy went to the store
  2. Chris did not go to the store and Sandy did not go to the store

- Do the two statements have the same meaning?

Example

- Let p represent “Chris went to the store”
- Let q represent “Sandy went to the store”

1. It is not the case that both Chris went to the store and Sandy went to the store:

$$\neg(p \wedge q)$$

2. Chris did not go to the store and Sandy did not go to the store:

$$(\neg p) \wedge (\neg q)$$

Example

p	q	$p \land q$	$\neg(p \land q)$

p	q	$\neg p$	$\neg q$	$(\neg p) \land (\neg q)$

Example

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T		
T	F		
F	T		
F	F		

p	q	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$

Example

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	
T	F	F	
F	T	F	
F	F	F	

p	q	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$

Example

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$

Example

p	q	$p \land q$	$\neg(p \land q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$(\neg p) \land (\neg q)$
T	T			
T	F			
F	T			
F	F			

Example

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

Example

p	q	$p \land q$	$\neg(p \land q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$(\neg p) \land (\neg q)$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Example

p	q	$p \land q$	$\neg(p \land q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$(\neg p) \land (\neg q)$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$\neg(p \land q)$ and $(\neg p) \land (\neg q)$ do not have the same meaning