Problem 2. [15 points]
Let p denote "You passed CS 2233".
Let q denote "You passed CS 3333".
Let r denote "You can register for CS 3343".
Let s denote "You understand propositional logic".

Use p, q, r, and s, to create propositions representing the following statements.

a. [5 points] You did not pass CS 2233, but you understand propositional logic.

$\neg p \wedge s$

b. [5 points] You cannot register for CS 3343 only if you have not passed both CS 2233 and CS 3333

$\neg r \rightarrow (\neg p \wedge \neg q)$

c. [5 points] If you can register for CS 3343, then you have passed CS 2233, and you understand propositional logic if you passed CS 2233

$(r \rightarrow p) \wedge (p \rightarrow s)$

Problem 3. [40 points]
Show that $\big(\neg q \wedge (p \vee p)\big) \rightarrow \neg q$ is a tautology, i.e. $\big(\neg q \wedge (p \vee p)\big) \rightarrow \neg q \equiv \mathrm{T}$
a. [10 points] By creating a truth table

p	q	$\neg q$	$p \vee p$	$\neg q \wedge (p \vee p)$	$\big(\neg q \wedge (p \vee p)\big) \rightarrow \neg q$
T	T	F	T	F	T
T	F	T	T	T	T
F	T	F	F	F	T
F	F	T	F	F	T

b. [10 points] By creating a sequence of logical equivalences and annotating each step

$\big(\neg q \wedge (p \vee p)\big) \rightarrow \neg q$	$\equiv \neg\big(\neg q \wedge (p \vee p)\big) \vee \neg q$	Conditional Identity
	$\equiv \big(\neg\neg q \vee \neg(p \vee p)\big) \vee \neg q$	De Morgan
	$\equiv \big(q \vee \neg(p \vee p)\big) \vee \neg q$	Double Negation
	$\equiv (\neg(p \vee p) \vee q) \vee \neg q$	Commutativity
	$\equiv \neg(p \vee p) \vee (q \vee \neg q)$	Associativity
	$\equiv \neg(p \vee p) \vee \mathrm{T}$	Negation
	$\equiv \mathrm{T}$	Domination

Show that $\neg q \rightarrow (p \wedge r) \equiv (\neg q \rightarrow r) \wedge (q \vee p)$
c. [10 points] By creating a truth table

p	q	r	$\neg q$	$(p \wedge r)$	$\neg q \rightarrow (p \wedge r)$	$\neg q \rightarrow r$	$q \vee p$	$(\neg q \rightarrow r) \wedge (q \vee p)$	$\neg q \rightarrow (p \wedge r) \leftrightarrow (\neg q \rightarrow r) \wedge (q \vee p)$
T	T	T	F	T	T	T	T	T	T
T	T	F	F	F	T	T	T	T	T
T	F	T	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T	F	T
F	T	T	F	F	T	T	T	T	T
F	T	F	F	F	T	T	T	T	T
F	F	T	T	F	F	T	F	F	T
F	F	F	T	F	F	F	F	F	T

d. [10 points] By creating a sequence of logical equivalences and annotating each step

$\neg q \to (p \wedge r)$	$\equiv \neg\neg q \vee (p \wedge r)$	Conditional Identity
	$\equiv (\neg\neg q \vee p) \wedge (\neg\neg q \vee r)$	Distributivity
	$\equiv (q \vee p) \wedge (\neg\neg q \vee r)$	Double Negation
	$\equiv (q \vee p) \wedge (\neg q \to r)$	Conditional Identity
	$\equiv (\neg q \to r) \wedge (q \vee p)$	Commutativity

Problem 4. [20 points]
a. [10 points] Show that the $\vee$ operator is associative by creating a truth table showing that $p \vee (q \vee r) \equiv (p \vee q) \vee r$.

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$p \vee q$	$(p \vee q) \vee r$	$p \vee (q \vee r) \leftrightarrow (p \vee q) \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	F	F	F	F	T

b. [10 points] The NOR operator $\downarrow$ is the negation of a disjunction: $p \downarrow q \equiv \neg(p \vee q)$. Its truth table is:

p	q	$p \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

Show that The NOR operator is not associative by creating a truth table showing that it is not the case that $p \downarrow (q \downarrow r) \equiv (p \downarrow q) \downarrow r$. In other words, create a truth table showing that $\big(p \downarrow (q \downarrow r)\big) \leftrightarrow \big((p \downarrow q) \downarrow r\big)$ is not a tautology.

p	q	r	$q \downarrow r$	$p \downarrow (q \downarrow r)$	$p \downarrow q$	$(p \downarrow q) \downarrow r$	$p \downarrow (q \downarrow r) \leftrightarrow (p \downarrow q) \downarrow r$
T	T	T	F	F	F	F	T
T	T	F	F	F	F	T	F
T	F	T	F	F	F	F	T
T	F	F	T	F	F	T	F
F	T	T	F	T	F	F	F
F	T	F	F	T	F	T	T
F	F	T	F	T	T	F	F
F	F	F	T	F	T	F	T