Section 3.1
Sets and Subsets

Sets

- A set is an unordered collection of distinct (no repetitions) members.

Ways to Describe Sets

- Sets can be described by listing their elements (members)
    - The set O of odd positive integers less than 10 is $\mathrm{O} = \{1, 3, 5, 7, 9\}$
- The elements of the set do not have to be the same type

$$S = \{22, \mathrm{hello}, \downarrow\}$$

- Ellipses can be used if it a clear pattern is established

$$T = \{1, 2, 3, \cdots, 100\}$$

is the set of positive integers less than or equal to 100

Set Membership

- When S is a set, the notation $x \in S$ means that x is an element of S
  - $3 \in \{1, 2, 3\}$

- The notation $x \notin S$ means that x is not an element of S
  - $4 \notin \{1, 2, 3\}$

Ways to Describe Sets

- Sets can also be described with the set builder notation
  - In general: $\{x \mid x \text{ has the property } P\}$
  - $\{x \mid x \text{ is an odd positive integer less than } 10\}$
    - "The set of xs such that x is an odd positive integer less than 10
  - $\left\{x \in \boldsymbol{R} \mid x = \frac{p}{q} \text{ for some positive integers } p \text{ and } q\right\}$
    - "The set of real numbers, x, such that $x = \frac{p}{q}$ for some positive integers p and q"
  - Note that : can be used instead of |., e.g., $\{n \in N\colon\ n > 100\}$

Ways to Describe Sets

- Abbreviations of common sets

$\boldsymbol{N}$	$= \{0, 1, 2, 3, \cdots\}$	The set of natural numbers
$\boldsymbol{Z}$	$= \{\cdots -2, -1, 0, 1, 2, \cdots\}$	The set of integers
$\boldsymbol{Z}^+$	$= \{1, 2, 3, \cdots\}$	The set of positive integers
$\boldsymbol{Q}$	$= \{p/q \mid p \in \boldsymbol{Z}, q \in \boldsymbol{Z} \text{ and } q \neq 0\}$	The set of rational numbers
$\boldsymbol{Q}^+$	$= \{p/q \mid p \in \boldsymbol{Z}^+, q \in \boldsymbol{Z}^+\}$	The set of positive rational numbers
$\boldsymbol{R}$		The set of real numbers
$\boldsymbol{R}^+$		The set of positive real numbers
$\boldsymbol{C}$		The set of complex numbers

Universal Set

- When working with sets, there is a universal set or domain of discourse
- The universal set is symbolized by U
- All sets are subsets of the universal set

Venn Diagrams

- Venn Diagrams are used to describe sets and elements
- Dots, labeled dots, or just labels represent elements
  - Unique elements are represented by single dots/labels
- Circles and other closed curves denote sets
  - They are often labeled
  - An overlap indicates the possibility of sets having elements in common
- The universal set is represented by a rectangle that contains everything

The Empty Set and Singleton Sets

- There is a set, called the empty set or the null set, with no elements
- The empty set can be written as ∅ or { }

$$\forall x \neg(x \in \emptyset)$$

- A set S that contains only one element is called a singleton set

$$\exists x\big(x \in S \land \forall y(y \in S \rightarrow x = y)\big)$$

Relations Between Sets

- Sets A and B are equivalent if they have the same members.

$A = B$ exactly when $\forall x (x \in A \leftrightarrow x \in B)$

$$\{1, 3, 5, 7, 9\} = \{x \mid x \in \boldsymbol{Z}^+, x \text{ is odd, and } x < 10\}$$

Relations Between Sets

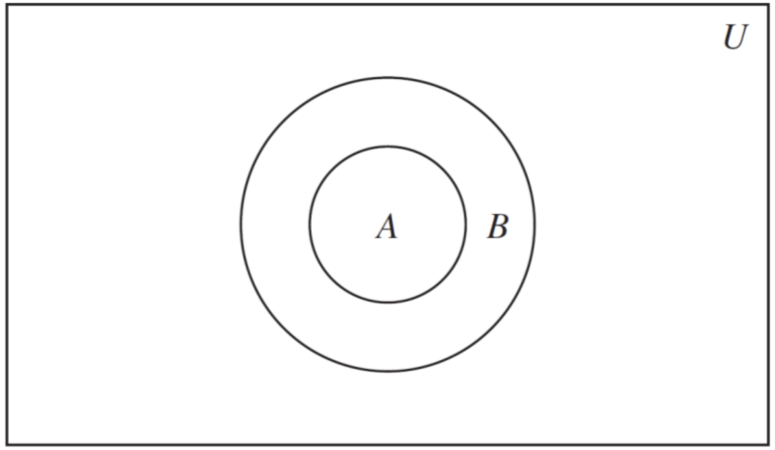
- Set A is a <u>subset</u> of set B if each element of A is also an element of B

$$A \subseteq B \text{ exactly when } \forall x(x \in A \rightarrow x \in B)$$

$$\{1, 2, 3\} \subseteq \boldsymbol{Z}$$

$$\emptyset \subseteq \{a, b\}$$

$$\emptyset \subseteq \emptyset$$

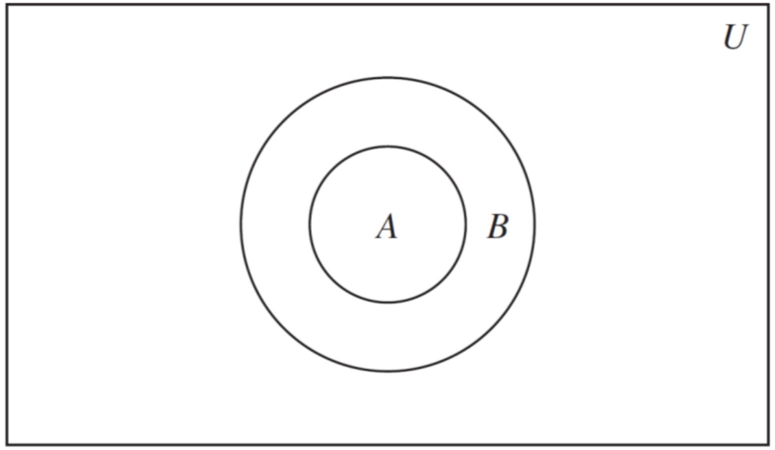


Relations Between Sets

- Set A is a <u>subset</u> of set B if each element of A is also an element of B

$$A \subseteq B \text{ exactly when } \forall x(x \in A \rightarrow x \in B)$$

- Venn diagram of $A \subseteq B$



Relations Between Sets

- $A \nsubseteq B$ is used to indicate that A is not a subset of B

$$\{1, 2, 3\} \nsubseteq \{1, 2\}$$

Relations Between Sets

- Set A is a <u>proper subset</u> of set of B if $A \subseteq B$ but $A \neq B$.
- The notation $A \subset B$ indicates that A is a proper subset of B

$$\{1, 2\} \subset \{1, 2, 3\}$$

- How can we express $A \subset B$ in predicate logic?

Cardinality

- If a set S has n members where n is a natural number, then S is a finite set and the cardinality (or size) of S is n

- The cardinality of a set S is denoted as $|S|$

$$|\{a, b, c\}| = 3$$

$$|\emptyset| = 0$$

- If a set S is not finite, then it is infinite