Homework Assignment 4
CS 2233
Sections 001 and 002
Due: 11:59pm Friday, March 1

Problem 1. [10 points]
Complete all participation activities in zyBook sections 2.7, 3.1-3.6, 4.1-4.3

Problem 2. [5 points]
Let $\max(x, y)$ be a function that returns the maximum of x and y, and let $\min(x, y)$ be a function that returns the minimum of x and y, Use a proof by cases to show that if $x, y \in \boldsymbol{R}$, then $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 3xy + y^2$

1. Assume $x, y \in \boldsymbol{R}$
2. Either $x \geq y$ or $x < y$
3. Case 1: Assume $x \geq y$
4. $\max(x, y) = x$ and $\min(x, y) = y$
5. $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = (\mathrm{x} + y)^2 + y \cdot x$
6. $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 2xy + y^2 + xy$
7. $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 3xy + y^2$
8. If $x \geq y$ then $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 3xy + y^2$
9. Case 2: Assume $x < y$
10. $\max(x, y) = y$ and $\min(x, y) = x$
11. $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = (y + x)^2 + x \cdot y$
12. $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = y^2 + 2yx + x^2 + xy$
13. $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 3xy + y^2$
14. If $x < y$ then $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 3xy + y^2$
15. Therefore, $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 3xy + y^2$

Problem 3. [10 points]
a. [5 points] Use the set builder notation to describe the set $\{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$.
$\{x \in \boldsymbol{Z} \mid x \geq -3 \text{ and } x \leq 5\}$

b. [5 points] Let $A = \{1, 4, 8, 16\}$ and $B = \{2, 4, 16, 32, 64\}$. Find $A \cup B$, $A \cap B$, $A - B$, $B - A$, and $|\mathcal{P}(A)|$.
$A \cup B = \{1, 2, 4, 8, 16, 32, 64\}$
$A \cap B = \{4, 16\}$
$A - B = \{1, 8\}$
$B - A = \{2, 32, 64\}$
$|\mathcal{P}(A)| = 2^4 = 16$

Problem 4. [10 points]
Prove $(A \cap B) \cup (A \cap \overline{B}) = A$

a) [5 points] By using a membership table

A	B	$\overline{B}$	$A \cap B$	$A \cap \overline{B}$	$(A \cap B) \cup (A \cap \overline{B})$
1	1	0	1	0	1
1	0	1	0	1	1
0	1	0	0	0	0
0	0	1	0	0	0

b) [5 points] By using set identities

$(A \cap B) \cup (A \cap \overline{B})$	$= A \cap (B \cup \overline{B})$	Distributive law
	$= A \cap U$	Complement law
	$= A$	Identity law

Problem 5. [15 points]
Determine whether each of these functions $f: \{a, b, c, d\} \to \{a, b, c, d\}$ is one-to-one (injection), and whether each of them is onto (surjection)

a. [5 points] $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
one-to-one and onto

b. [5 points] $f(a) = b, f(b) = b, f(c) = d, f(d) = c$
neither one-to-one nor onto

c. [5 points] $f(a) = d, f(b) = b, f(c) = c, f(d) = d$
neither one-to-one nor onto

Problem 6. [15 points]
Determine whether each of these functions $f: \boldsymbol{R} \to \boldsymbol{R}$ is a one-to-one correspondence (i.e., onto and one-to-one)

a. [5 points] $f(x) = -3x + 4$
one-to-one correspondence

b. [5 points] $f(x) = -3x^2 + 7$
not a one-to-one-correspondence (for each x, $f(x) \leq 7$)

c. [5 points] $f(x) = (x + 2)(x - 1)x$
not a one-one-correspondence ($f(-2) = f(1) = f(0) = 0$)

Problem 7. [15 points]
Recall that $\boldsymbol{N} = \{0, 1, 2, 3, ...\}$. Give an example of a function from $\boldsymbol{N}$ to $\boldsymbol{N}$ that is:

a. [5 points] one-to-one but not onto
$f(x) = x + 1$

b. [5 points] onto but not-one-to-one
$f(x) = \lfloor x/2 \rfloor$

c. [5 points] neither one-to-one nor onto
$f(x) = 0$

(Hint: consider using the absolute value, floor, or ceiling functions for part b)