Section 3.2
Sets of Sets

Power Set of Sets

- It is possible for a set to contains sets as elements

$$S = \left\{ \{a, b\}, \quad \{b, c, d\}, \quad \emptyset \right\}$$

Power Sets

- If S is a set, then the power set of S is the set that contains exactly all subsets of S.
- The power set of S is denoted as $\mathcal{P}(S)$
  - $\mathcal{P}(\emptyset) = \{\emptyset\}$
  - $\mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$
  - $\mathcal{P}(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Power Sets

- If $|S| = n$, then $|\mathcal{P}(S)| = 2^n$
- If a set S has n members, then there are 2^n different subsets of S

Section 3.3
Union and Intersection

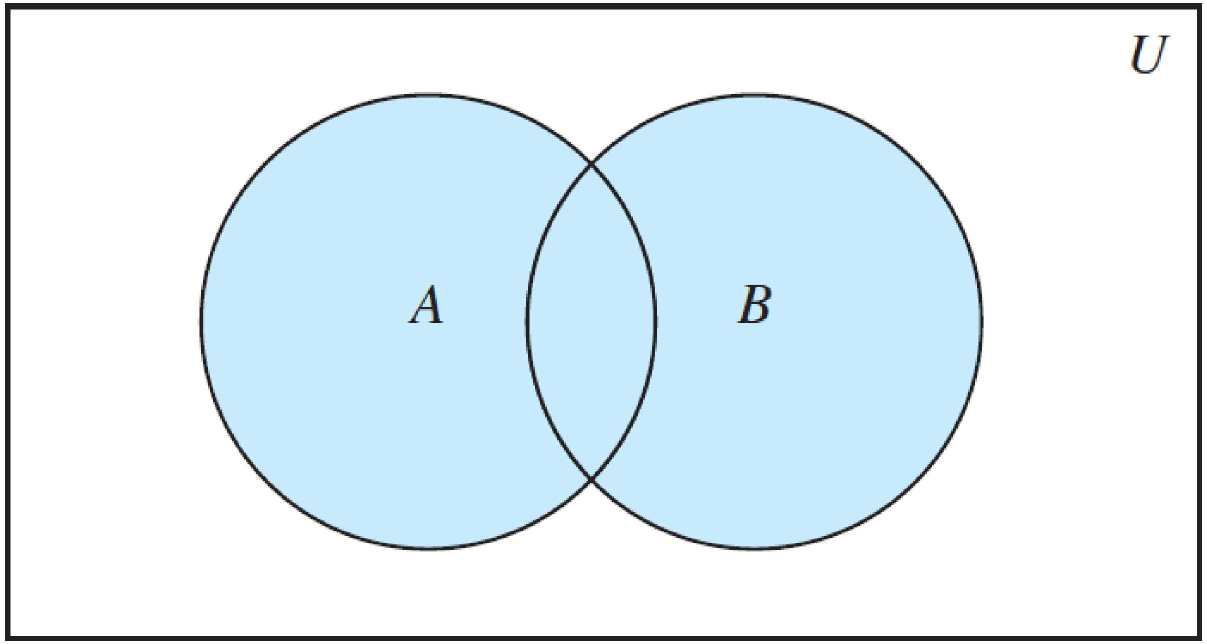
Set Union

- If A and B are sets, then the union of A and B is the set containing exactly those elements that are in A, in B, or in both

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$\{1, 3, 5\} \cup \{1, 2, 3\} = \{1, 2, 3, 5\}$$

Set Union



$A \cup B$ is shaded.

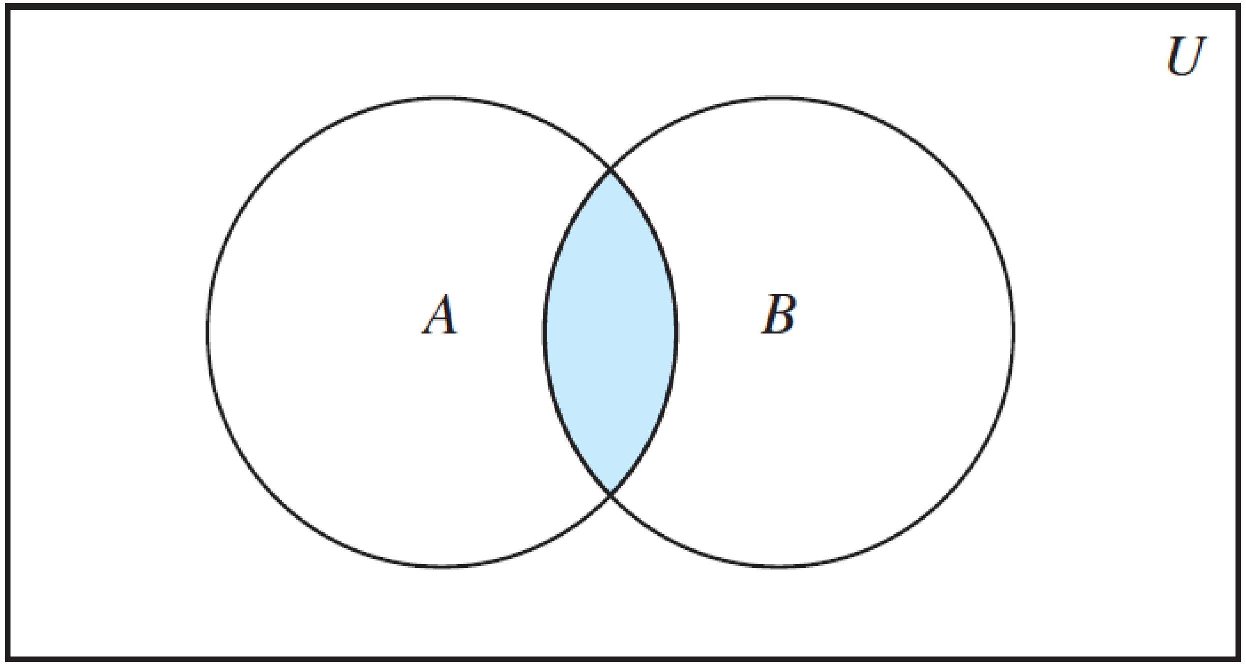
Set Intersection

- If A and B are sets, then the intersection of A and B is the set containing exactly those elements that are both in A and in B

$$A \cap B = \{x \mid x \in A \land x \in B\}$$

$$\{1, 3, 5\} \cap \{1, 2, 3\} = \{1, 3\}$$

Set Intersection



$A \cap B$ is shaded.

Example

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

$$C = \{2, 3, 5, 7\}$$

$$A \cup (B \cap C) = \{1, 2, 3, 4,5\}$$

Intersections of Sequences of Sets

Let $A_1, A_2, \ldots, A_n$ be a sequence of sets

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

$$= \{a \mid a \in A_i \text{ for each } i, 1 \leq i \leq n\}$$

Intersections of Sequences of Sets

Example:

$$A_1 = \{1, 2, 3, 4, 5\}$$

$$A_2 = \{2, 3, 5, 7\}$$

$$A_3 = \{1, 2, 4, 5, 8, 9\}$$

$$\bigcap_{i=1}^{3} A_i = \{2, 5\}$$

Unions of Sequences of Sets

Let $A_1, A_2, \ldots, A_n$ be a sequence of sets

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$$

$$= \{a \mid a \in A_i \text{ for some } i, 1 \leq i \leq n\}$$

Unions of Sequences of Sets

Example:

$$A_1 = \{1, 2, 3\}$$

$$A_2 = \{3, 5, 7\}$$
$$A_3 = \{2, 4\}$$

$$\bigcup_{i=1}^{3} A_i = \{1, 2, 3, 4, 5, 7\}$$