Section 8.1
Sequences

Informal Sequences

- Informally a sequence is an ordered list of objects from a set
  - a, b, c, …
  - 1, 4, 9, 16, …
- Since the sequence is ordered, it has a first element, second element, third element, etc.

Informal Sequences

- In general, we can list a sequence using subscripts to indicate order:
  - $a_1, a_2, a_3, \cdots$
  - $b_0, b_1, b_2, \cdots$
  - Often starting with 1 or 0
- Because of the ordering, we can think of a function f from a subset of the integers (to another set) that when given an integer i produces the member of a sequence that is at position i

$$f(i) = a_i$$

Sequences

- A <u>sequence</u> is a function from a subset of the integers to a set S
- The domain of the sequence is usually $\{0, 1, 2, 3, \cdots\}$ or $\{1, 2, 3, \cdots\}$ but it can be any set of consecutive integers
- When function f is a sequence, a_n denotes $f(n)$
  - a_n is called a <u>term</u> of the sequence
  - The notation $\{a_n\}_{n \in \boldsymbol{N}}$ denotes the entire sequence $a_0, a_1, a_2, \cdots$
  - The notation $\{a_n\}_{n \in \boldsymbol{Z}^+}$ denotes the entire sequence $a_1, a_2, a_3, \cdots$

Sequence Examples

- Example: Let $\{a_n\}_{n \in \boldsymbol{Z}^+}$ be a sequence where $a_n = \frac{1}{n}$
- This sequence starts with $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$

- We can also describe this sequence as $\{1/n\}_{n \in \boldsymbol{Z}^+}$

Sequence Examples

- Example: Let $\{a_n\}_{n \in \boldsymbol{N}}$ be a sequence where $a_n = 2n$
- This sequence starts with $0, 2, 4, 6, \cdots$

- We can also describe this sequence as $\{2n\}_{n \in \boldsymbol{N}}$

Geometric Progressions

- A geometric progression is a sequence of the form:

$$a, ar, ar^2, \cdots ar^n, \cdots$$

where the initial term a and common ratio r are real numbers

- How could we describe a geometric progression using the compact $\{\quad\}$ notation?

Geometric Progressions

- We can express

$$a, ar, ar^2, \cdots ar^n, \cdots$$

as

$$ar^0, ar^1, ar^2, \cdots ar^n, \cdots$$

and hence

$$\{ar^n\}_{n \in \boldsymbol{N}}$$

Geometric Progressions

- Example 1: The sequence

$$1,\ -1,\ 1,\ -1,\ 1, \cdots$$

is a geometric progression with initial term 1 and common ratio -1

$$1(-1)^0, 1(-1)^1, 1(-1)^2, 1(-1)^3, \cdots$$

Geometric Progressions

- Example 2: The sequence

$$2,\ 10,\ 50,\ 250,\ 1250, \cdots$$

is a geometric progression with initial term 2 and common ratio 5

$$2(5)^0, 2(5)^1, 2(5)^2, 2(5)^3, \cdots$$

Geometric Progressions

- Example 3: The sequence

$$6,\ 2,\ \frac{2}{3},\ \frac{2}{9},\ \frac{2}{27}, \cdots$$

is a geometric progression with initial term 6 and common ratio 1/3

$$6\left(\frac{1}{3}\right)^{0}, 6\left(\frac{1}{3}\right)^{1}, 6\left(\frac{1}{3}\right)^{2}, 6\left(\frac{1}{3}\right)^{3}, \cdots$$

Arithmetic Progression

- An arithmetic progression is a sequence of the form:

$$a, a + d, a + 2d, \cdots a + nd, \cdots$$

where the initial term a and common difference d are real numbers

- How could we describe an arithmetic progression using the compact $\{\quad\}$ notation?

Arithmetic Progressions

- We can express

$$a, a + d, a + 2d, \cdots a + nd, \cdots$$

as

$$a + 0(d), a + 1(d), a + 2(d), \cdots a + n(d), \cdots$$

and hence

$$\{a + nd\}_{n \in \boldsymbol{N}}$$

Arithmetic Progressions

- Example 1: The sequence

$$-1,\ 3,\ 7,\ 11,\ 15, \cdots$$

is an arithmetic progression with initial term -1 and common difference 4

$$-1 + 0(4), -1 + 1(4), -1 + 2(4), -1 + 3(4), \cdots$$

Arithmetic Progressions

- Example 2: The sequence

$$7,\ 4,\ 1,\ -2,\ -5, \cdots$$

is an arithmetic progression with initial term 7 and common difference -3

$$7+0(-3), 7+1(-3), 7+2(-3), 7+3(-3), \cdots$$

Sequences

- Example: A students puts \$1000 in a bank account and each year earns 10% interest. Show that the amount of money in the account each year is a geometric progression

Sequences

- Example: A students puts $1000 in a bank account and each year earns 10% interest. Show that the amount of money in the account each year is a geometric progression
- The first year there is $\$1000$ in the account
- The second year there is $\$1000 + 0.1(\$1000) = \$1100$ in the account
- The third year there is $\$1100 + 0.1(\$1100) = \$1210$ in the account

Sequences

- Earning 10% interest means that if there is $\$X$ in the bank account in a given year, then there is $\$X + 0.1(\$X) \ = \ 1.1(\$X)$ in the account the next year
- And $1.1(1.1)(\$X) = 1.1^2(\$X)$ the year after that
- The sequence of amounts of money in the account is

$$\$1000 \ \ 1.1(\$1000)\ 1.1^2(\$1000)\ 1.1^3(\$1000) \ \ \ldots$$

- This is a geometric progression with initial value 1000 and common ratio 1.1