Section 13.2
Graph Representations

Representing Graphs

- Programs that work with graphs need a way to represent them
- Depending on the goal of the program, different representations can be used
- Sets of edges can be represented in different ways
- For each vertex v in a graph, an adjacency list is a list of vertices that are adjacent to v in the graph

Adjacency Lists

- Example: Give the adjacency lists for each vertex in the following undirected graph

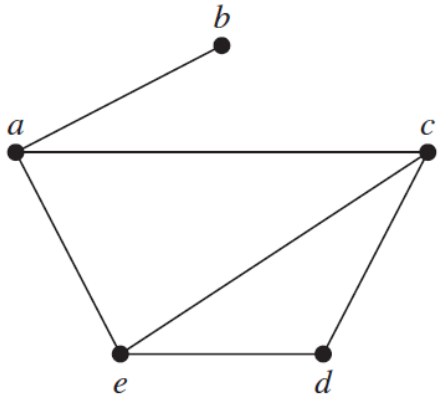


FIGURE 1 **A simple graph.**

TABLE 1 **An Adjacency List for a Simple Graph.**

Vertex	*Adjacent Vertices*
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

Adjacency Lists

- Example: Give the adjacency lists for each vertex in the following directed graph

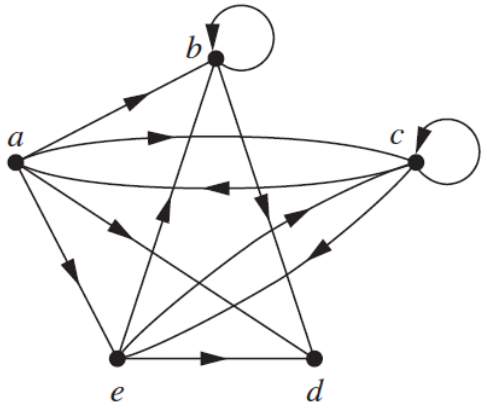


FIGURE 2 **A directed graph.**

TABLE 2 **An Adjacency List for a Directed Graph.**

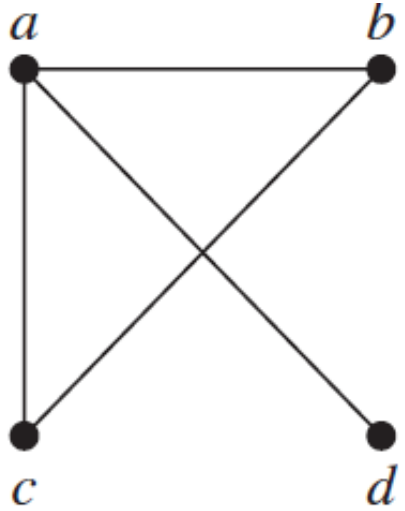
Initial Vertex	*Terminal Vertices*
a	b, c, d, e
b	b, d
c	a, c, e
d	
e	b, c, d

Adjacency Matrices

- The edges of a graph can also be represented by an adjacency matrix
- An adjacency matrix is a table containing 1s and 0s that describes the edges in a graph
- The value at row i and column j is a 1 if there is an edge connecting vertex i and vertex j
- The value at row i and column j is a 0 if there is no edge connecting vertex i and vertex j

Adjacency Matrices

- Example: Give the adjacency matrix for the following undirected graph.



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Adjacency matrices for undirected graphs are symmetric: The entry at row i and column j is the same as the entry at row j and column i

Adjacency Matrices

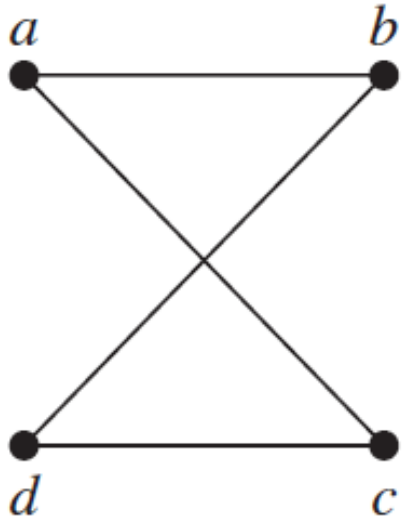
- Example 4: Draw an undirected graph from the following adjacency matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Adjacency Matrices

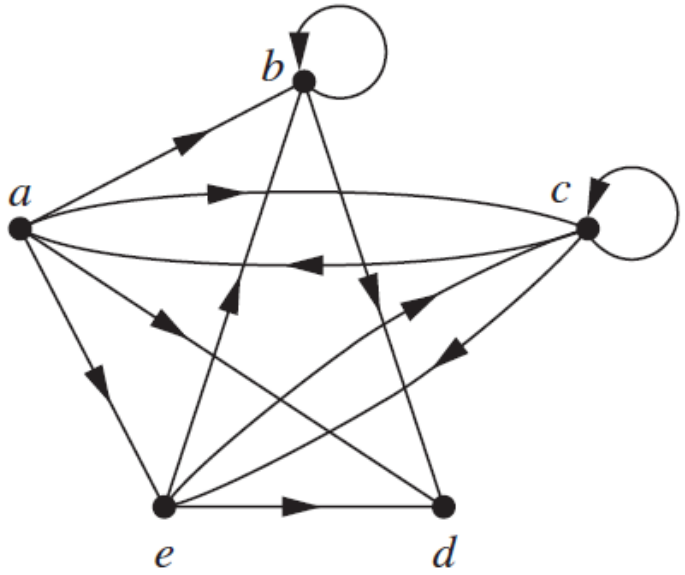
- Example : Draw an undirected graph from the following adjacency matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



Adjacency Matrices

- Example: Give the adjacency matrix for the following directed graph



Terminal vertices

Initial vertices

	a	b	c	d	e
a	0	1	1	1	1
b	0	1	0	1	0
c	1	0	1	0	1
d	0	0	0	0	0
e	0	1	1	1	0

Adjacency Matrices

- Example: Draw a directed graph from the following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Adjacency Matrices

- Example: Draw a directed graph from the following adjacency matrix

Terminal vertices

Initial vertices

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Adjacency Matrices

- Example: Draw a directed graph from the following adjacency matrix

Terminal vertices

Initial vertices

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Adjacency Matrices

- Example: Draw a directed graph from the following adjacency matrix

Terminal vertices

Initial vertices

	a	b	c	d
a	0	1	1	0
b	0	0	0	0
c	0	0	1	1
d	1	1	0	0