Homework Assignment 8
CS 2233
Section 001 and Section 002
Due: Friday, April 19

Problem 1. [10 points]
Complete all participation activities in zyBook sections 8.13-8.15, 8.17

Problem 2. [10 points] Solve the following recurrence relations.

a. [5 points]

$$a_0 = 1$$
$$a_1 = 1$$
$$a_n = 2a_{n-1} + 15a_{n-2}$$

Assume $a_n = r^n$

$r^n = 2r^{n-1} + 15r^{n-2}$
$r^n - 2r^{n-1} - 15r^{n-2} = 0$
$r^2 - 2r - 15 = 0$
$(r-5)(r+3) = 0$
$r = 5, -3$

$a_n = s5^n + t(-3)^n$

$a_0 = 1 = s5^0 + t(-3)^0 = s + t$ and hence $s = 1 - t$
$a_1 = 1 = s5^1 + t(-3)^1 = s5 - t3$ and hence $5s - 3t = 1$

$5s - 3t = 1$
$5(1-t) - 3t = 1$
$5 - 5t - 3t = 1$
$4 = 8t$
$t = 1/2$
$s = 1/2$

$$a_n = \frac{1}{2}5^n + \frac{1}{2}(-3)^n$$

b. [5 points]

$$f_0 = 1$$
$$f_1 = 4$$
$$f_n = 2f_{n-1} - f_{n-2}$$

Assume $f_n = r^n$

$r^n = 2r^{n-1} - r^{n-2}$
$r^n - 2r^{n-1} + r^{n-2} = 0$
$r^2 - 2r + 1 = 0$
$(r-1)(r-1) = 0$
$r = 1, 1$

$f_n = s1^n + tn1^n$

$f_0 = 1 = s1^0 + t(0)1^0 = s$ and hence $s = 1$
$f_1 = 4 = s1^1 + t(1)1^1 = s + t$ and hence $s + t = 4$

$s + t = 4$

$1 + t = 4$
$t = 3$
$s = 1$

$f_n = 1^n + 3n1^n$ or $f_n = 1 + 3n$

Problem 3. [15 points] Use the Master Theorem to give big-Θ estimates of the following recurrence relations.

a. [5 points] $T(1) = 1$; $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2, d = 2$
$a/b^d = 4/2^d = 1$
$T(n)$ is $\Theta\left(n^2 log(n)\right)$

b. [5 points] $T(1) = 1$; $T(n) = 9T(n/3) + n$

$a = 9, b = 3, d = 1$
$a/b^d = 9/3^1 > 1$
$T(n)$ is $\Theta\left(n^{log_3(9)}\right)$ i.e., $T(n)$ is $\Theta(n^2)$

c. [5 points] $T(1) = 1$; $T(n) = 6T(n/2) + n^3$

$a = 6, b = 2, d = 3$
$a/b^d = 6/2^3 < 1$
$T(n)$ is $\Theta(n^3)$