Compound Propositions - Conditional

- Also known as "implication"
- $p \rightarrow q$
- Truth table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- "if p then q", "p implies q", "p only if q", "p is sufficient for q", "q is necessary for p", "q unless $\neg p$"

Compound Propositions - Conditional

- Assume p represents “You will get 100% on the final exam”
- and q represents “You will get an 'A' for the course”
- Then $p \rightarrow q$ represents
  - “If you will get 100% on the final exam then you will get an 'A' for the course.”
  - Note: nothing is guaranteed if you do not get 100% on the final; you could still get an 'A'.

Compound Propositions - Conditional

- Assume p represents “John wakes up early”
- and q represents “John will be late for the meeting”
- Then $(\neg p) \rightarrow q$ represents
  - "If John does not wake up early, then John will be late for the meeting"

Converse, Contrapositive, and Inverse

- The converse of $p \rightarrow q$ is $q \rightarrow p$
- The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$
- The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$

Compound Propositions - Biconditional

- Also known as "bi-implication" or "double implication"
- $p \leftrightarrow q$
- Truth table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- "p if and only if q"

Truth Tables for Compound Propositions

- Example: $(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
- Since there are two variables, p and q, create a table with four rows and columns for sub-propositions

p	q	$\neg q$	$p \leftrightarrow q$	$p \vee \neg q$	$(p \leftrightarrow q) \rightarrow (p \vee \neg q)$

Truth Tables for Compound Propositions

- Example: $(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
- List all possible combinations of truth values for p and q

p	q	$\neg q$	$p \leftrightarrow q$	$p \vee \neg q$	$(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
T	T				
T	F				
F	T				
F	F				

Truth Tables for Compound Propositions

- Example: $(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
- Complete the $\neg q$ column using the q column

p	q	$\neg q$	$p \leftrightarrow q$	$p \vee \neg q$	$(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
T	T	F			
T	F	T			
F	T	F			
F	F	T			

Truth Tables for Compound Propositions

- Example: $(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
- Complete the $p \leftrightarrow q$ column using the p and q columns

p	q	$\neg q$	$p \leftrightarrow q$	$p \vee \neg q$	$(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
T	T	F	T		
T	F	T	F		
F	T	F	F		
F	F	T	T		

Truth Tables for Compound Propositions

- Example: $(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
- Complete the $p \vee \neg\, q$ column using the p and $\neg q$ columns

p	q	$\neg q$	$p \leftrightarrow q$	$p \vee \neg q$	$(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
T	T	F	T	T	
T	F	T	F	T	
F	T	F	F	F	
F	F	T	T	T	

Truth Tables for Compound Propositions

- Example: $(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
- Complete the $(p \leftrightarrow q) \rightarrow (p \vee \neg q)$ column using the $p \leftrightarrow q$ and $p \vee \neg q$ columns

p	q	$\neg q$	$p \leftrightarrow q$	$p \vee \neg q$	$(p \leftrightarrow q) \rightarrow (p \vee \neg q)$
T	T	F	T	T	T
T	F	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T

Example

- Create a truth table for $(p \wedge q) \vee (r \wedge s)$

Example

p	q	r	s			$(p \wedge q) \vee (r \wedge s)$

Example

p	q	r	s	$(p \land q)$	$(r \land s)$	$(p \land q) \lor (r \land s)$

Example

p	q	r	s	$(p \wedge q)$	$(r \wedge s)$	$(p \wedge q) \vee (r \wedge s)$
T	T	T	T			
T	T	T	F			
T	T	F	T			
T	T	F	F			
T	F	T	T			
T	F	T	F			
T	F	F	T			
T	F	F	F			
F	T	T	T			
F	T	T	F			
F	T	F	T			
F	T	F	F			
F	F	T	T			
F	F	T	F			
F	F	F	T			
F	F	F	F			

Example

p	q	r	s	$(p \wedge q)$	$(r \wedge s)$	$(p \wedge q) \vee (r \wedge s)$
T	T	T	T	T		
T	T	T	F	T		
T	T	F	T	T		
T	T	F	F	T		
T	F	T	T	F		
T	F	T	F	F		
T	F	F	T	F		
T	F	F	F	F		
F	T	T	T	F		
F	T	T	F	F		
F	T	F	T	F		
F	T	F	F	F		
F	F	T	T	F		
F	F	T	F	F		
F	F	F	T	F		
F	F	F	F	F		

Example

p	q	r	s	$(p \wedge q)$	$(r \wedge s)$	$(p \wedge q) \vee (r \wedge s)$
T	T	T	T	T	T	
T	T	T	F	T	F	
T	T	F	T	T	F	
T	T	F	F	T	F	
T	F	T	T	F	T	
T	F	T	F	F	F	
T	F	F	T	F	F	
T	F	F	F	F	F	
F	T	T	T	F	T	
F	T	T	F	F	F	
F	T	F	T	F	F	
F	T	F	F	F	F	
F	F	T	T	F	T	
F	F	T	F	F	F	
F	F	F	T	F	F	
F	F	F	F	F	F	

Example

p	q	r	s	$(p \wedge q)$	$(r \wedge s)$	$(p \wedge q) \vee (r \wedge s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	T
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	F	F
T	F	F	T	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	T	F	F	F	F
F	T	F	T	F	F	F
F	T	F	F	F	F	F
F	F	T	T	F	T	T
F	F	T	F	F	F	F
F	F	F	T	F	F	F
F	F	F	F	F	F	F

Tautologies, Contradictions, and Contingencies

- If a proposition is true for all values of its variables, then it is a tautology.
  - $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Tautologies, Contradictions, and Contingencies

- If a proposition is false for all values of its variables, then it is a <u>contradiction</u>
  - $p \land \neg p$

p	$\neg p$	$p \land \neg p$
T	F	F
F	T	F

Tautologies, Contradictions, and Contingencies

- If a proposition is neither a tautology nor a contradiction, then it is called a <u>contingency</u>
  - $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical Equivalences

- Two propositions, p and q, are logically equivalent if $p \leftrightarrow q$ is a tautology.
- The notation $p \equiv q$ denotes that p and q are logically equivalent.
  - Sometimes $p \Leftrightarrow q$ is used instead of $p \equiv q$
- Note that $p \equiv q$ is not a proposition. It is a statement about $p \leftrightarrow q$ being a tautology

Logical Equivalences

- Example: $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$p \rightarrow q \leftrightarrow \neg p \vee q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

De Morgan's Laws

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

De Morgan's Laws

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

p	q	$p \land q$	$\neg(p \land q)$	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$\neg(p \land q) \leftrightarrow \neg p \lor \neg q$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

De Morgan's Laws

$$\neg(p \lor q) \equiv \neg p \land \neg q$$

p	q	$p \lor q$	$\neg(p \lor q)$	$\neg p$	$\neg q$	$\neg p \land \neg q$	$\neg(p \lor q) \leftrightarrow \neg p \land \neg q$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

De Morgan's Laws

- De Morgan's laws can be applied to cases with compound propositions as well as propositional variables. Substitute the same compound proposition for all occurrences of the same variable

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Substitute $s \to t$ for p and $p \wedge q$ for q

$$\neg\big((s \to t) \vee (p \wedge q)\big) \equiv \neg(s \to t) \wedge \neg(p \wedge q)$$

De Morgan's Laws and Natural Language

- Use De Morgan's laws to express the negation of:

  "Miguel has a cellphone and he has a laptop computer"
- Let p represent "Miguel has a cellphone"
- Let q represent "Miguel has a laptop computer"
- $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- "Miguel does not have a cellphone or he does not have a laptop computer"

Examples of Using De Morgan's Laws

- $\neg\big((p \vee (r \to q)) \wedge (r \vee s)\big)$

Examples of Using De Morgan's Laws

- $\neg\big((p \vee (r \rightarrow q)) \wedge (r \vee s)\big)$

- $\neg\big(p \vee (r \rightarrow q)\big) \vee \neg(r \vee s)$

Examples of Using De Morgan's Laws

- $\neg\big((p \vee s) \vee r\big) \wedge \neg(p \vee q)$

Examples of Using De Morgan's Laws

- $\neg\big((p \vee s) \vee r\big) \wedge \neg(p \vee q)$

- $\neg\Big(\big((p \vee s) \vee r\big) \vee (p \vee q)\Big)$

Examples of Using De Morgan's Laws

- $\neg\big((\neg r \rightarrow s) \wedge (\neg p \vee r)\big) \vee \neg\big((s \wedge \neg t) \vee (r \rightarrow p)\big)$

Examples of Using De Morgan's Laws

- $\neg\big((\neg r \to s) \land (\neg p \lor r)\big) \lor \neg\big((s \land \neg t) \lor (r \to p)\big)$

- $\neg\Big(\big((\neg r \to s) \land (\neg p \lor r)\big) \land \big((s \land \neg t) \lor (r \to p)\big)\Big)$