Homework Assignment 4
CS 2233
Sections 001 and 002
Due: 11:59pm Friday, March 1

Problem 1. [10 points]
Complete all participation activities in zyBook sections 2.7, 3.1-3.6, 4.1-4.3

Problem 2. [5 points]
Let $\max(x, y)$ be a function that returns the maximum of x and y, and let $\min(x, y)$ be a function that returns the minimum of x and y, Use a proof by cases to show that if $x, y \in \boldsymbol{R}$, then $(\max(x, y) + \min(x, y))^2 + \min(x, y) \cdot \max(x, y) = x^2 + 3xy + y^2$

Problem 3. [10 points]
a. [5 points] Use the set builder notation to describe the set $\{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$.
b. [5 points] Let $A = \{1, 4, 8, 16\}$ and $B = \{2, 4, 16, 32, 64\}$. Find $A \cup B$, $A \cap B$, $A - B$, $B - A$, and $|\mathcal{P}(A)|$.

Problem 4. [10 points]
Prove $(A \cap B) \cup \left(A \cap \overline{B}\right) = A$

a) [5 points] By using a membership table
b) [5 points] By using set identities

Problem 5. [15 points]
Determine whether each of these functions $f: \{a, b, c, d\} \to \{a, b, c, d\}$ is one-to-one (injection), and whether each of them is onto (surjection)

a. [5 points] $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
b. [5 points] $f(a) = b, f(b) = b, f(c) = d, f(d) = c$
c. [5 points] $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

Problem 6. [15 points]
Determine whether each of these functions $f: \boldsymbol{R} \to \boldsymbol{R}$ is a one-to-one correspondence (i.e., onto and one-to-one)

a. [5 points] $f(x) = -3x + 4$
b. [5 points] $f(x) = -3x^2 + 7$
c. [5 points] $f(x) = (x + 2)(x - 1)x$

Problem 7. [15 points]
Recall that $\boldsymbol{N} = \{0, 1, 2, 3, \ldots\}$. Give an example of a function from $\boldsymbol{N}$ to $\boldsymbol{N}$ that is:

a. [5 points] one-to-one but not onto
b. [5 points] onto but not-one-to-one
c. [5 points] neither one-to-one nor onto

(Hint: consider using the absolute value, floor or ceiling functions for part b)