Section 6.1
Introduction to Binary Relations

Review: Cartesian Product (Section 3.6)

- The cartesian product of two sets A and B, $A \times B$, is the set containing all of the ways that a member of A can be paired with a member of B.

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

- Note that (a, b) is an ordered pair (2-tuple)

Binary Relations

- Let A and B be sets (possibly the same set). A binary relation from A to B is a subset of $A \times B$

- If R is a binary relation, then the following are synonyms for $(a, b) \in R$
  - aRb
  - $R(a, b)$
  - "a is related to b by R"

Binary Relations

- Example 1: Let A be a set of students and B be a set of courses. Let $R = \{(a, b) | \ a \in A, b \in B, \text{and student } a \text{ is enrolled in course } b\}$

  If students Jason and Deborah are both enrolled in CS 2233, then

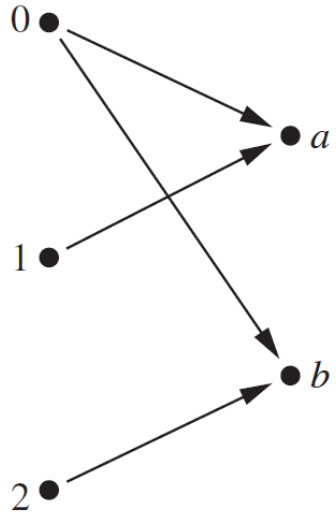
  - $(\text{Jason}, \text{CS } 2233) \in R$
  - $(\text{Deborah}, \text{CS } 2233) \in R$

Binary Relations

- Example 2: Let A be the set of cities in the USA and B be the set of states in the USA. Let $R = \{(a, b) | \text{ city } a \text{ is in state } b\}$
  - $(\text{San Antonio}, \text{Texas}) \in R$
  - $(\text{Boulder}, \text{Colorado}) \in R$

Binary Relations

- Example 3: Let $A = \{0, 1, 2\}$ and $B = \{a, b\}$. Let R be the following relation from A to B: $R = \{(0, a), (0, b), (1, a), (2, b)\}$
  - R can be displayed graphically or with a table



R	a	b
0	1	1
1	1	0
2	0	1

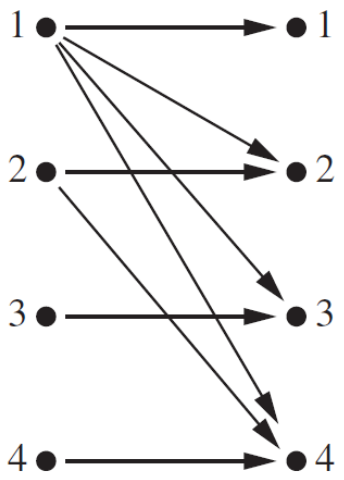
Relations on a Set

- A relation on a set A is a relation from A to A
  - A relation on a set A is a subset of $A \times A$
  - The set A is the domain of the relation

Relations on a Set

- Example 4: Let $A = \{1, 2, 3, 4\}$ and R be the relation on A where $(a, b) \in R$ if and only if a evenly divides b, then:

$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$



R	1	2	3	4
1	1	1	1	1
2	0	1	0	1
3	0	0	1	0
4	0	0	0	1

Relations on a Set

- Example 6: How many relations are there on a set with n elements?
  - Each subset of $A \times A$ is a relation on A
  - $A \times A$ has n^2 elements
  - $A \times A$ has 2^{n^2} subsets
    - The set $\{1, 2, 3\}$ has $2^{3^2} = 2^9 = 512$ different relations