Homework Assignment 7
CS 2233
Section 001 and Section 002
Due: Friday, April 12

Problem 1. [10 points]
Complete all participation activities in zyBook sections 8.6-8.11

Problem 2. [20 points] Consider a proof by strong induction on the set $\{12, 13, 14, \ldots\}$ of $\forall n\ P(n)$ where $P(n)$ is: n cents of postage can be formed by using only 3-cent stamps and 7-cent stamps

a. [5 points] For the base case, show that $P(12)$, $P(13)$, and $P(14)$ are true
b. [5 points] What is the induction hypothesis?
c. [5 points] What do you need to prove for the inductive step?
d. [5 points] Complete the inductive step for $k + 3$ cents of postage

Problem 3. [5 points] Prove by using strong induction on the positive integers $\forall n P(n)$ where $P(n)$ is: The positive integer n can be expressed as the sum of different powers of 2

For example, $19 = 16 + 2 + 1 = 2^4 + 2^1 + 2^0$

Hint: For the inductive step, separately consider the cases where $k + 1$ is even and odd. When $k + 1$ is even, $(k + 1)/2$ is an integer.

Problem 4. [10 points] Let S be a set of ordered pair of integers defined recursively as follows.
1. $(0, 0) \in S$
2. If $(a, b) \in S$, then $(a + 1, b + 3) \in S$ and $(a + 3, b + 1) \in S$
3. Nothing else is in S

a. [5 points] List the elements in S that result from applying the recursive rule 0, 1, 2, and 3 times
b. [5 points] Use structural induction to show that for all $(a, b) \in S$, $a + b$ is a multiple of 4.

Problem 5. [15 points] Write down the first 6 elements of the following sequences where $n \in \{1, 2, 3 \ldots\}$ and then give a recursive definition for a_n. For part c, express the first 6 elements as powers of 2.

a. [5 points] $a_n = 3n - 10$
b. [5 points] $a_n = (1 + (-1)^n)^n$
c. [5 points] $a_n = 2^{n!}$